

Ontologies/ontology languages further extend the expressiveness:

- Description Logics
- Topic Maps (in SGML) since early 90s, XTM (XML Topic Maps)
- Ontolingua – non-XML approach from the Knowledge Representation area
- OIL (Ontology Inference Layer): initiative funded by the EU programme for Information Society Technologies (project: On-To-Knowledge, 1.2000-10.2002); based on RDF/RDFS
- DAML (Darpa Agent Markup Language; 2000) ... first ideas for a Semantic Web language
- DAML+OIL (Jan. 2001)
- developed into OWL (1st version March 02, finalized Feb. 04)

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THREE VARIANTS OF OWL

Several expressiveness/complexity/decidability levels:

- OWL Full: extension of RDF/RDFS
 - classes can also be regarded as individuals (have properties, classes of classes etc.)
- OWL DL
 - fragment of OWL that fits into the [Description Logics](#) Framework:
 - * the sets of classes, properties, individuals and literals are disjoint
 - ⇒ only individuals can have arbitrary user-specified properties;
classes and properties have only properties from the predefined RDFS and OWL vocabularies.
 - decidable reasoning
 - OWL 1.0 (2004), OWL 2.0 (2009)
- OWL Lite
 - subset of OWL DL
 - easier migration from frame-based tools (note: F-Logic is a frame-based framework)
 - easier reasoning (translation to Datalog)

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7.1 Description Logics

- Focus on the description of *concepts*, not of instances
- Terminological Reasoning
- Origin of DLs: Semantic Networks (graphical formalism)

Notions

- Concepts (= classes),
note: literal datatypes (string, integer etc.) are not classes in DL and OWL, but *data ranges*
(cf. XML Schema: distinction between `simpleTypes` and `complexType`)
- Roles (= relationships),
- A Description Logic alphabet consists of a finite set of concept names (e.g. Person, Cat, LivingBeing, Male, Female, ...) and a finite set of role names (e.g., hasChild, marriedTo, ...),
- constructors for derived concepts and roles,
- axioms for asserting facts about concepts and roles.

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COMPARISON WITH OTHER LOGICS

Syntax and semantics defined different but similar from first-order logic

- formulas over an alphabet and a small set of additional symbols and combinators
- semantics defined via *interpretations* of the combinators
- set-oriented, no instance variables
(FOL: instance-oriented with domain quantifiers)
- family of languages depending on what combinators are allowed.

The base: \mathcal{AL}

The usual starting point is \mathcal{AL} :

- “attributive language”
- Manfred Schmidt-Schauss and Gert Smolka: *Attributive Concept Descriptions with Complements*. In *Artificial Intelligence* 48(1), 1991, pp. 1–26.
- extensions (see later: \mathcal{ALC} , \mathcal{ALCQ} , $\mathcal{ALCQ}(D)$, \mathcal{ALCQI} , \mathcal{ALCN} etc.)

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ATOMIC, NAMED CONCEPTS

- atomic concepts, e.g., Person, Male, Female
- the “universal concept” \top (often called “Thing” – everything is an instance of Thing)
- the empty concept \perp (“Nothing”). There is no thing that is an instance of \perp .

CONCEPT EXPRESSIONS USING SET OPERATORS

- intersection of concepts: $A \sqcap B$
Adult \sqcap Male
- negation: $\neg A$
 \neg Italian , Person \sqcap \neg Italian
- union (disjunctive concept): $A \sqcup B$
Cat \sqcup Dog – things where it is known that they are cats or dogs, but not necessarily which one.

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CONCEPT EXPRESSIONS USING ROLES

Concepts (as an intensional characterization of sets of instances) can be described implicitly by their properties (wrt. *roles*).

Let R be a role, C a concept. Then, the expressions $\exists R.C$ and $\forall R.C$ also describe concepts (intensionally defined concepts) by constraining the roles:

- Existential quantification: $\exists R.C$ – all things that have a *filler* for the role R that is in C .
 \exists hasChild.Male means “all things that have a male child”.
Syntax: the whole expression is the “concept expression”, i.e., \exists hasChild.Male(*john*) stands for $(\exists$ hasChild.Male)(*john*).
- Range constraints: $\forall R.C$
 \forall hasChild.Male means “all things that have only male children (including those that have no children at all)”.
- Note that \perp can be used to express non-existence: $\forall R.\perp$: all things where all fillers of role R are of the concept \perp (= Nothing) – i.e., all things that do not have a filler for the role R .
 \forall hasChild. \perp means “all things that have no children”.

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SEMANTICS OF CONCEPT CONSTRUCTORS

As usual: by interpretations.

An interpretation $\mathcal{I} = (\mathcal{I}, \mathcal{D})$ consists of the following:

- a domain \mathcal{D} ,
- for every concept name C : $I(C) \subseteq \mathcal{D}$ is a subset of the domain,
- for every role name R : $I(R) \subseteq \mathcal{D} \times \mathcal{D}$ is a binary relation over the domain.

Structural Induction

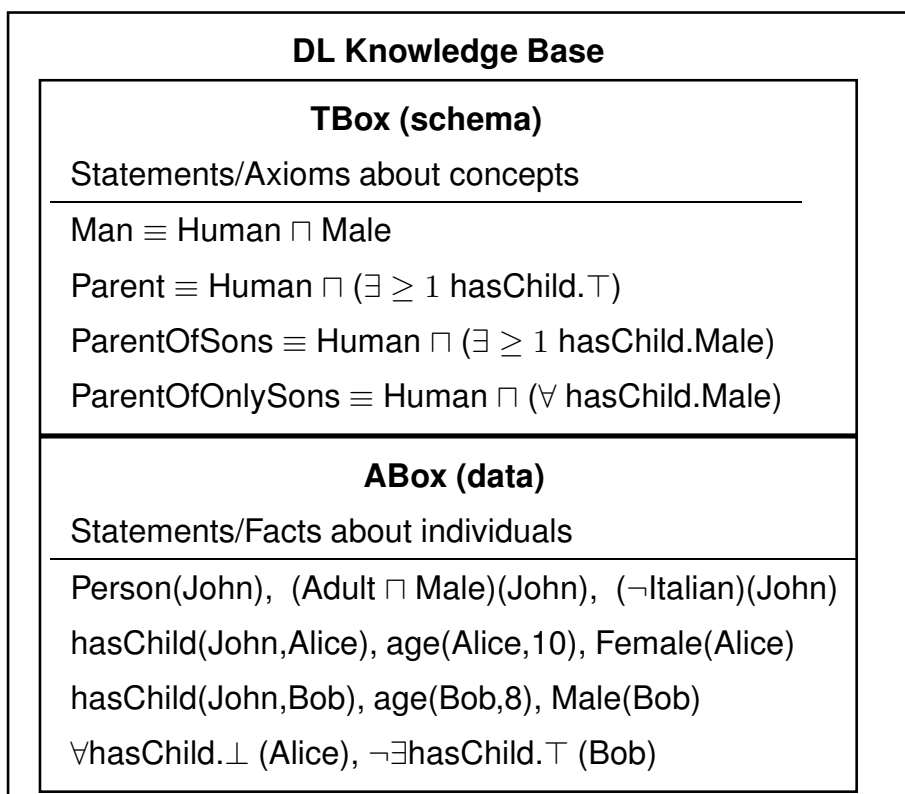
- $I(A \sqcup B) = I(A) \cup I(B)$
- $I(A \sqcap B) = I(A) \cap I(B)$
- $I(\neg A) = \mathcal{D} \setminus I(A)$
- $I(\exists R.C) = \{x \mid \text{there is an } y \text{ such that } (x, y) \in I(R) \text{ and } y \in I(C)\}$
- $I(\forall R.C) = I(\neg \exists R.(\neg C)) = \{x \mid \text{for all } y \text{ such that } (x, y) \in I(R), y \in I(C)\}$

Example

$\text{Male} \sqcap \forall \text{hasChild.Male}$ is the set of all men who have only sons.

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STRUCTURE OF A DL KNOWLEDGE BASE



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THE TBOX: TERMINOLOGICAL AXIOMS

Definitions and assertions (not to be understood as constraints) about concepts:

- concept subsumption: $C \sqsubseteq D$; defining a concept hierarchy.
Semantics: $\mathcal{I} \models C \sqsubseteq D \iff I(C) \subseteq I(D)$.
- concept equivalence: $C \equiv D$; often used for defining the left-hand side concept.
Semantics: $\mathcal{I} \models C \equiv D \iff C \sqsubseteq D$ and $D \sqsubseteq C$.

TBox Reasoning

- is a concept C satisfiable?
- is $C \sqsubseteq D$ implied by a TBox
- given the definition of a new concept D , classify it wrt. the given concept hierarchy.

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THE ABOX: ASSERTIONAL AXIOMS

- contains the facts about instances (using names for the instances) in terms of the basic concepts and roles:
`Person(John), Male(John), hasChild(John,Alice)`
- contains also knowledge in terms of intensional concepts, e.g., $\exists \text{hasChild.Male(John)}$

TBox + ABox Reasoning

- check consistency between ABox and a given TBox
- ask whether a given instance satisfies a concept C
- ask for all instances that have a given property
- ask for the most specific concepts that an instance satisfies

Note: instances are allowed only in the ABox, not in the TBox.

If instances should be used in the definition of concepts (e.g., “European Country” or “Italian City”), *Nominals* must be used (see later).

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FAMILY OF DL LANGUAGES UP TO \mathcal{ALC}

- \mathcal{AL} : intersection, negation of *atomic* concepts
- \mathcal{AL} : restricted existential quantification: $\exists R.\top$
 $\exists \text{hasChild}.\top$ means “all things that have a child (... that belongs to the concept “Thing”)”.
- \mathcal{AL} has no “branching” (no union, or any kind of disjunction); so proofs in \mathcal{AL} are linear.
- \mathcal{U} : “union”; e.g. $\text{Parent} \equiv \text{Father} \sqcup \text{Mother}$.
- \mathcal{C} : negation (“complement”) of non-atomic concepts.
 $\text{Person} \sqcap \neg \exists \text{hasChild}.\top$ characterizes the set of persons who have no children (note: open-world semantics of negation!)
 Note: the FOL equivalent would be expressed via variables:
 $\forall x(\text{Childless}(x) \leftrightarrow (\text{Person}(x) \wedge \neg \exists y(\text{hasChild}(x, y))))$
- \mathcal{U} and \mathcal{E} can be expressed by \mathcal{C} .
- \mathcal{ALC} is the “smallest” Description Logic that is closed wrt. the set operations.
- A frequently used restriction of \mathcal{AL} is called \mathcal{FL}^- (for “Frame-Language”), which is obtained by disallowing negation completely (i.e., having only positive knowledge).

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FAMILY OF DL LANGUAGES: EXTENSIONS TO \mathcal{ALC}

- \mathcal{E} : (unrestricted) existential quantification of the form $\exists R.C$ (recall that \mathcal{AL} allows only $\exists R.\top$).
 $\exists \text{hasChild}.\text{Male}$, for persons who have at least one male child,
 $\exists \text{hasChild}.\text{hasChild}.\top$ for grandparents.
 Note: the FOL equivalent uses variables:
 $p(x) \leftrightarrow \exists y(\text{hasChild}(x, y) \wedge \text{Male}(y)),$
 $p(x) \leftrightarrow \exists y(\text{hasChild}(x, y) \wedge \exists x : \text{hasChild}(y, x)).$
- Exercise: show why unrestricted existential quantification $\exists R.C$ in contrast to $\exists R.\top$ leads to branching.
- \mathcal{N} : (unqualified) cardinalities of roles (“number restrictions”).
 $(\geq 3 \text{ hasChild}.\top)$ for persons who have at least 3 children.
- \mathcal{Q} : qualified role restrictions:
 $(\leq 2 \text{ hasChild}.\text{Male})$
 \mathcal{F} : like \mathcal{Q} , but restricted to cardinalities 0, 1 and “arbitrary”.

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COMPLEXITY AND DECIDABILITY: OVERVIEW

- Logic \mathcal{L}^2 , i.e., FOL with only two (reusable) variable symbols is decidable.
- Full FOL is undecidable.
- DLs: incremental, modular set of semantical notions.
- only part of FOL is required for concept reasoning.
- \mathcal{ALC} can be *expressed* by FOL, but then, the inherent semantics is lost \rightarrow full FOL reasoner required.
- Actually, \mathcal{ALC} can be encoded in FOL by only using two variables \rightarrow \mathcal{ALC} is decidable.
- Consistency checking of \mathcal{ALC} -TBoxes and -ABoxes is PSPACE-complete (proof by reduction to *Propositional Dynamic Logic* which is in turn a special case of propositional multimodal logics).
There are algorithms that are efficient in the average case.
- \mathcal{ALCN} goes beyond \mathcal{L}^2 and PSPACE. Reduction to \mathcal{C}^2 (including “counting” quantifiers) yields decidability, but now in NEXPTIME. There are algorithms for \mathcal{ALCN} and even \mathcal{ALCQ} in PSPACE.

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FURTHER EXTENSIONS

- Role hierarchy (\mathcal{H} ; role subsumption and role equivalence, union/intersection of roles):
 $\text{hasSon} \sqsubseteq \text{hasChild}$, $\text{hasChild} \equiv \text{hasSon} \sqcup \text{hasDaughter}$
- *Role Constructors* similar to regular expressions:
concatenation ($\text{hasGrandchild} \equiv \text{hasChild} \circ \text{hasChild}$), transitive closure
($\text{hasDescendant} \equiv \text{hasChild}^+$) (indicated by e.g. \mathcal{ALCH}_{reg}), and inverse
($\text{isChildOf} \equiv \text{hasChild}^-$) (\mathcal{I}).
- *Data types* (indicated by “(D)”), e.g. integers.
 $\text{Adult} \equiv \text{Person} \sqcap \exists \text{age}. \geq 18$.
- *Nominals* (\mathcal{O}) allow to use individuals from the ABox also in the TBox.
Enumeration Concepts: $\text{BeNeLux} \equiv \{\text{Belgium, Netherlands, Luxemburg}\}$,
HasValue-Concepts: $\text{GermanCity} \equiv \exists \text{inCountry.Germany}$.
- *Role-Value-Maps*:
Equality Role-Value-Map: $(R_1 \equiv R_2)(x) \Leftrightarrow \forall y : R_1(x, y) \leftrightarrow R_2(x, y)$.
Containment Role-Value-Map: $(R_1 \sqsubseteq R_2)(x) \equiv \forall y : R_1(x, y) \rightarrow R_2(x, y)$.
 $(\text{knows} \sqsubseteq \text{likes})$ describes the set of people who like all people they know;
i.e., $(\text{knows} \sqsubseteq \text{likes})(\text{John})$ denotes that John likes all people he knows.

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FORMAL SEMANTICS OF EXPRESSIONS

- $I(\geq nR.C) = \{x \mid \#\{y \mid (x, y) \in I(R) \text{ and } y \in I(C)\} \geq n\}$,
- $I(\leq nR.C) = \{x \mid \#\{y \mid (x, y) \in I(R) \text{ and } y \in I(C)\} \leq n\}$,
- $I(nR.C) = \{x \mid \#\{y \mid (x, y) \in I(R) \text{ and } y \in I(C)\} = n\}$,
- $I(R \sqcup S) = I(R) \cup I(S)$, $I(R \sqcap S) = I(R) \cap I(S)$,
- $I(R \circ S) = \{(x, z) \mid \exists y : (x, y) \in I(R) \text{ and } (y, z) \in I(S)\}$,
- $I(R^-) = \{(y, x) \mid (x, y) \in I(R)\}$,
- $I(R^+) = (I(R))^+$.
- If nominals are used, \mathcal{I} also assigns an element $I(x) \in \mathcal{D}$ to each nominal symbol x (similar to constant symbols in FOL). With this,
 $I(\{x_1, \dots, x_n\}) = \{I(x_1), \dots, I(x_n)\}$, and
 $I(R.y) = \{x \mid \{z \mid (x, z) \in I(R)\} = \{y\}\}$,
- $I(R_1 \equiv R_2) = \{x \mid \forall y : R_1(x, y) \leftrightarrow R_2(x, y)\}$,
 $I(R_1 \sqsubseteq R_2) = \{x \mid \forall y : R_1(x, y) \rightarrow R_2(x, y)\}$.

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OVERVIEW: COMPLEXITY OF EXTENSIONS

- \mathcal{ALC}_{reg} , $\mathcal{ALCHI}Q_{\mathcal{R}^+}$, \mathcal{ALCIO} are ExpTime-complete, $\mathcal{ALCHI}QO_{\mathcal{R}^+}$ is NExpTime-Complete.,
- Combining *composite* roles with cardinalities becomes undecidable (encoding in FOL requires 3 variables).
- Encoding of Role-Value Maps with composite roles in FOL is undecidable (encoding in FOL requires 3 variables; the logic loses the *tree model property*).
- $\mathcal{ALC}QI_{reg}$ with role-value maps restricted to boolean compositions of *basic* roles remains decidable. Decidability is also preserved when role-value-maps are restricted to functional roles.

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DESCRIPTION LOGIC MODEL THEORY

The definition is the same as in FOL:

- an interpretation is a model of an ABox A if
 - for every atomic concept C and individual x such that $C(x) \in A$, $I(x) \in I(C)$, and
 - for every atomic role R and individuals x, y such that $R(x, y) \in A$, $(I(x), I(y)) \in I(R)$.
- note: the interpretation of the non-atomic concepts and roles is given as before,
- all axioms ϕ of the TBox are satisfied, i.e., $\mathcal{I} \models \phi$.

Based on this, DL entailment is also defined as before:

- a set Φ of formulas entails another formula Ψ (denoted by $\Phi \models \psi$), if $\mathcal{I}(\Psi) = \text{true}$ in all models \mathcal{I} of Φ .

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DECIDABILITY, COMPLEXITY, AND ALGORITHMS

Many DLs are decidable, but in high complexity classes.

- decidability is due to the fact that often *local* properties are considered, and the verification proceeds tree-like through the graph without connections between the branches.
- This locality does not hold for cardinalities over composite roles, and for role-value maps – these lead to undecidability.
- Reasoning algorithms for \mathcal{ALC} and many extensions are based on tableau algorithms, some use model checking (finite models), others use tree automata.

Three types of Algorithms

- restricted (to polynomial languages) and complete
- expressive logics with complete, worst-case EXPTIME algorithms that solve realistic problems in “reasonable” time. (Fact, Hermit, Racer, Pellet)
- more expressive logics with incomplete reasoning.

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EXAMPLE

- Given facts: $\text{Person} \equiv \text{Male} \sqcup \text{Female}$ and $\text{Person}(\text{unknownPerson})$.
- Query $?-\text{Male}(X)$ yields an empty answer
- Query $?-\text{Female}(X)$ yields an empty answer
- Query $?-(\text{Male} \sqcup \text{Female})(X)$ yields unknownPerson as an answer
- for query answering, *all* models of the $\text{TBox}+\text{ABox}$ are considered.
- in some models, the unknownPerson is Male, in the others it is female.
- in all models it is in $(\text{Male} \sqcup \text{Female})$.

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SUMMARY AND COMPARISON WITH FOL

Base Data (DL atomic concepts and atomic roles \sim RDF)

- unary predicates (concepts/classes): $\text{Person}(\text{John})$,
- binary predicates (roles/properties): $\text{hasChild}(\text{John}, \text{Alice})$

Expressions

Concept/Role Expressions act as unary/binary predicates:

- $(\exists \text{hasChild}.\text{Male})(\text{John})$, $(\text{Adult} \sqcap \text{Parent})(\text{John})$,
- $(\text{hasChild} \circ \text{hasChild})(\text{Jack}, \text{Alice})$, $(\text{neighbor}^*)(\text{Portugal}, \text{Germany})$

\Rightarrow disjunction, conjunction and quantifiers *only* in the restricted contexts of expressions

\Rightarrow implications *only* in the restricted contexts of TBox Axioms:

- $C_1 \sqsubseteq C_2$ $\text{Parent} \sqsubseteq \text{Person}$ • $R_1 \sqsubseteq R_2$ $\text{capital} \sqsubseteq \text{hasCity}$
- $C_1 \equiv C_2$ $\text{Parent} \equiv \exists \text{hasChild}.\top$ • $R_1 \equiv R_2$ $\text{neighbor} \equiv (\text{neighbor} \sqcup \text{neighbor}^-)$

\Rightarrow ABox/TBox (=database) is a conjunctive set of atoms.

\Rightarrow No formulas!

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7.2 OWL

- the OWL versions use certain DL semantics:
- Base: $\mathcal{ALC}_{\mathcal{R}^+}$: (i.e., with transitive roles). This logic is called \mathcal{S} (reminiscent to its similarity to the modal logic S).
- roles can be ordered hierarchically (`rdfs:subPropertyOf`; \mathcal{H}).
- OWL Lite: $\mathcal{SHIF}(D)$, Reasoning in EXPTIME.
- OWL DL: $\mathcal{SHOIN}(D)$, decidable.
Pellet (2007) implements $\mathcal{SHOIQ}(D)$. Decidability is in NEXPTIME (combined complexity wrt. TBox+ABox), but the actual complexity of a given task is constrained by the maximal used cardinality and use of nominals and inverses and behaves like the simpler classes.
(Ian Horrocks and Ulrike Sattler: A Tableau Decision Procedure for SHOIQ(D); In IJCAI, 2005, pp. 448-453; available via <http://dblp.uni-trier.de>)
- OWL 2.0 towards $\mathcal{SROIQ}(D)$ and more datatypes ...