

$(\text{brother of } \circ \text{ child}) \sqsubseteq \text{uncle of}$
 $\therefore X \xrightarrow{\text{brother of}} \cdot \downarrow \text{child} \therefore Y$
 $\therefore X \text{ uncle of } \therefore Y$
 $\therefore X \xrightarrow{\text{married}} \cdot \xrightarrow{\text{sister of}} \cdot \downarrow \text{child}$
 $((\text{married } \circ \text{ sister of}) \circ \text{ child}) \sqsubseteq \text{uncle } \therefore Y \text{ of}$
 $\text{Uncle of More} \equiv \geq 2 \text{ uncle of } \cdot \top$
 $\text{Cyclic} \equiv \text{Node } \cap \{x \mid x \text{ to } x\}$

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$\emptyset \models \text{uncle}(\text{john}, \text{barbara})$ proven
 $\emptyset \not\models \neg \text{uncle}(\text{john}, \text{barbara})$
 $\forall y: (\neg \text{brother of}(\text{john}, y) \vee \neg \text{child}(y, \text{barbara}))$
[forall y] $\neg \text{brother}(\text{john}, y) \vee \neg \text{child}(y, \text{barbara})$
 $\neg \text{brother}(\text{john}, y) \quad \neg \text{child}(y, \text{barbara})$
 $\square y \rightarrow \text{sue} \quad \neg \text{child}(\text{sue}, \text{barbara})$
 \square

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Tableau über $\Phi = \phi_1 \wedge \dots \wedge \phi_n$

Schemata

ϕ_1
 \vdots
 ϕ_2
 \vdots
 ϕ_n

\leftarrow look into each of the formulas

Tableau Rules for First order-logic

ϕ_1
 \vdots
 ϕ_2
 \vdots
 ϕ_n
 $A \wedge B$
 \vdots
 A
 B

\vdots
 $A \vee B$
 \vdots
 A B

$\exists x: \phi(x)$
 $\exists y: \phi(y)$
 $\phi(a)$
 $\phi(b)$

$\forall x \phi(x)$
 \vdots
 $\phi(X)$

x a FOL variable
 X is a tableau var.

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FOL example

Knowledge Base Φ

recall:

 $A \rightarrow B \equiv \neg A \vee B$
 $\neg \exists z: q(z) \equiv \forall z \neg q(z)$
 $\dots \rightarrow \neg q(z)$

$\forall x: p(x) \rightarrow q(x)$
 $\exists y: p(y)$
 $\neg \exists z: q(z)$
 $p(a)$
 $p(X) \rightarrow q(X)$
 $\neg p(X)$
 $\neg q(X)$
 $q(a)$
 $\neg q(z)$
 $\Box z \rightarrow a$

want to show:
 $\exists z q(z)$
 claim:
 $\Phi \vdash \exists z: q(z)$
 Tableau:
 Φ
 $\neg \exists z: q(z)$
 try to close

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$\forall x \exists y : \text{father}(x, y)$

$\forall x \exists y : p(x, y)$

$\exists y : p(X, y)$

$p(X, f(X))$

y is a function of x

$\begin{array}{cc} \text{---} & \text{---} \\ \text{D } X \rightarrow \text{john} & p(\text{john}, f(\text{john})) \end{array}$

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DL formulas

kinds of formulas :

$C \equiv (\dots)$

$C \sqsubseteq D$

$C \sqsubseteq \forall \text{child. Person}$

~~Person(john)~~

child(john, bob)

Sons

Two children Parent $\equiv \geq 2$ child. Male

Two sons parent(Sue)

$\Rightarrow 2$ child. Male(Sue)

Male(a)

Male(b)

child(Sue, b)

child(Sue, a)

$a \neq b$

$\rightsquigarrow a, b$
both male
both children of Sue

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$(\bigcap D) (s)$
 \vdots
 $C(s)$
 $D(s)$

$(C \cup D) (s)$
 $\swarrow \quad \searrow$
 $C(s) \quad D(s)$

$(\exists R.C) (s)$
 \downarrow
 $R(s, a)$
 $C(a)$

$(\forall R.C) (s)$
 \downarrow
 $R(s, t)$
 $C(t)$

$(\geq n R.C) (s)$

$R(s, a_1)$

$C(a_1)$

\vdots

$R(s, a_n)$

$C(a_n)$

forall $a_i \neq a_j$

$R(s, a) - \text{Male}(a) \wedge \neg \text{Ad}(a)$

$R(s, b) - \text{Female}(b) \wedge \neg \text{Ad}(b)$

$R(s, c) - \text{Male}(c) \wedge \neg \text{Ad}(c)$

$(\leq 2 R.T) (s)$

$a=b \rightarrow \text{Male}(b) \quad \square$

$a=c \rightarrow \text{Ad}(c) \quad \square$

$b=c \rightarrow \text{Female}(c) \quad \square$

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?- Male(X)

Answer: john

DB

$\neg \text{Male}(X)$

\vdots

Male(john)

$\square X \rightarrow \text{john}$

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