Chapter 7
Ontologies and the Web Ontology Language – OWL

- *vocabularies* can be defined by RDFS
  - not so much stronger than the ER Model or UML
    (even weaker: no cardinalities)
  - not only a conceptual model, but a “real language” with a close connection to the data level (RDF)
  - *incremental* world-wide approach
  - “global” vocabulary can be defined by autonomous partners
- but: still restricted when *describing* the vocabulary.

Ontologies/ontology languages further extend the expressiveness:

- Description Logics
- Topic Maps (in SGML) since early 90s, XTM (XML Topic Maps)
- Ontolingua – non-XML approach from the Knowledge Representation area
- OIL (Ontology Inference Layer): initiative funded by the EU programme for Information Society Technologies (project: On-To-Knowledge, 1.2000-10.2002); based on RDF/RDFS
- DAML (Darpa Agent Markup Language; 2000) ... first ideas for a Semantic Web language
- DAML+OIL (Jan. 2001)
- developed into OWL (1st version March 02, finalized Feb. 04)

THREE VARIANTS OF OWL

Several expressiveness/complexity/decidability levels:

- OWL Full: extension of RDF
  - classes can also be regarded as individuals (classes of classes ... higher-order reasoning)
- OWL DL
  - fragment of OWL that fits into the Description Logics Framework
  - decidable reasoning
- OWL Lite
  - subset of OWL DL
  - easier migration from frame-based tools
    (note: F-Logic was a frame-based framework)
  - easier reasoning

7.1 Description Logics

- Focus on the description of *concepts*, not of instances
- Terminological Reasoning
- Origin of DLs: Semantic Networks (graphical formalism)

Notions

- Concepts (= classes),
  note: literal datatypes (string, integer etc.) are not classes in DL and OWL, but *data ranges*
  (cf. XML Schema: distinction between simpleTypes and complexTypes)
- Roles (= relationships),
- A Description Logic alphabet consists of a finite set of concept names (e.g. Person, Cat, LivingBeing, Male, Female, . . .) and a finite set of role names (e.g., hasChild, marriedTo, . . .),
- constructors for drived concepts and roles,
- axioms for asserting facts about concepts and roles.

COMPARISON WITH OTHER LOGICS

Syntax and semantics defined different but similar from first-order logic

- formulas over an alphabet and a small set of additional symbols and combinators
- semantics defined via *interpretations* of the combinators
- set-oriented, no instance variables
  (FOL: instance-oriented with domain quantifiers)
- family of languages depending on what combinators are allowed.

The base: $\mathcal{AL}$

The usual starting point is $\mathcal{AL}$:

- "attributive language"
- Manfred Schmidt-Schauss and Gert Smolka: *Attributive Concept Descriptions with Complements*. In *Artificial Intelligence* 48(1), 1991, pp. 1–26.
- extensions (see later: $\mathcal{ALC}$, $\mathcal{ALCQ}$, $\mathcal{ALCQ}(D)$, $\mathcal{ALCQI}$, $\mathcal{ALCN}$ etc.)

ATOMIC, NAMED CONCEPTS

- atomic concepts, e.g., Person, Male, Female
- the "universal concept" $\top$ (often called "Thing" – everything is an instance of Thing)
- the empty concept $\bot$ ("Nothing"). There is no thing that is an instance of $\bot$.

SET OPERATIONS

- intersection of concepts: $A \sqcap B$
- negation: $\neg A$
  $\mathcal{AL}$ allows only atomic negation.
- union: $A \sqcup B$
  Union is not allowed in $\mathcal{AL}$.

INTENSIONAL CONCEPTS

Concepts (as an intensional characterization of sets of instances) can be described implicitly by their properties (wrt. *roles*).

Let R be a role, C a concept. Then, the expressions $\exists R.C$ and $\forall R.C$ also dscribe concepts (intensionally defined concepts) by constraining the roles:

- Existential quantification: $\exists R.C$ – all things that have a *filler* for the role R that is in C.
  ∃hasChild.Male describes all things that have a male child.
- $\mathcal{AL}$: only as restricted existential quantification: $\exists R.\top$
  ∃hasChild.⊤ describes all things that have a child (formally: that belongs to the concept "Thing").
- Range constraints: $\forall R.C$
  ∀hasChild.Male describes all things that have only male children (including those that have no children at all).
- Note that $\bot$ can be used to express non-existence: $\forall R.\bot$ describes all things where all fillers of role R are of the concept $\bot$ (= Nothing) – i.e., all things that do not have a filler for the role R.
  ∀hasChild.⊥ describes the things that have no children.

Semantics of Concept Constructors

As usual: by interpretations.

An interpretation $\mathcal{I}$ consists of the following:

- a domain $\mathcal{D}$,
- for every concept name C: $C^{\mathcal{I}} \subseteq \mathcal{D}$ is a subset of the domain,
- for every role name R: $R^{\mathcal{I}} \subseteq \mathcal{D} \times \mathcal{D}$ is a binary relation over the domain.

Structural Induction

- $(A \sqcup B)^{\mathcal{I}} = A^{\mathcal{I}} \cup B^{\mathcal{I}}$
- $(A \sqcap B)^{\mathcal{I}} = A^{\mathcal{I}} \cap B^{\mathcal{I}}$
- $(\neg A)^{\mathcal{I}} = \mathcal{D} \setminus A^{\mathcal{I}}$
- $(\exists R.C)^{\mathcal{I}} = \{x \mid$ there is an y such that $(x, y) \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\}$
- $(\forall R.C)^{\mathcal{I}} = \{x \mid$ for all y such that $(x, y) \in R^{\mathcal{I}}$, $y \in C^{\mathcal{I}}\}$

Example

Male $\sqcap$ $\forall$hasChild.Male is the set of all men who have only sons.

Structure of a DL Knowledge Base

DL Knowledge Base

TBox (schema)

Talks about concepts

Man $\equiv$ Human $\sqcap$ Male

Parent $\equiv$ Human $\sqcap$ ($\exists \geq 1$ hasChild.$\top$)

ParentOfSons $\equiv$ Human $\sqcap$ ($\exists \geq 1$ hasChild.Male)

ParentOfOnlySons $\equiv$ Human $\sqcap$ ($\forall$ hasChild.Male)

ABox (data)

Talks about individuals

Person(John), Male(John)

hasChild(John,Alice), age(Alice,10), Female(Alice)

hasChild(John,Bob), age(Bob,8), Male(Bob)

The TBox: Terminological Axioms

Definitions and assertions (not to be understood as constraints) about concepts:

- concept subsumption: $C \sqsubseteq D$; defining a concept hierarchy. $\mathcal{I} \models C \sqsubseteq D \quad :\Leftrightarrow \quad C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.
- concept equivalence: $C \equiv D$; often used for defining the left-hand side concept. Semantics: $\mathcal{I} \models C \equiv D \quad :\Leftrightarrow \quad C \sqsubseteq D$ and $D \sqsubseteq C$.
- analogous for role subsumption and role equivalence.

TBox Reasoning

- is a concept C satisfiable?
- is $C \sqsubseteq D$ implied by a TBox
- given the definition of a new concept D, classify it wrt. the given concept hierarchy.

THE ABOX: ASSERTIONAL AXIOMS

- contains the facts about instances (using names for the instances) in terms of the basic concepts and roles:
  Person(John), Male(John), hasChild(John,Alice)
- contains also knowledge in terms of intensional concepts, e.g., ∃hasChild.Male(John)

TBox + ABox Reasoning

- check consistency between ABox and a given TBox
- ask whether a given instance satisfies a concept C
- ask for all instances that have a given property
- ask for the most specific concepts that an instance satisfies

Note: instances are allowed only in the ABox, not in the TBox.

If instances should be used in the definition of concepts (e.g., "European Country" or "Italian City"), *Nominals* must be used (see later).

EXTENSIONS TO $\mathcal{AL}$

- $\mathcal{U}$: "union"; e.g. Parent $\equiv$ Father $\sqcup$ Mother.
- $\mathcal{C}$: negation ("complement") of non-atomic concepts.
  Person $\sqcap \neg\exists$hasChild.$\top$ characterizes the set of persons who have no children (note: open-world semantics of negation!)

  Note: the FOL equivalent would be expressed via variables:
  $\forall x(\text{Childless}(x) \leftrightarrow (\text{Person}(x) \wedge \neg\exists y(\text{hasChild}(x, y))))$
- $\mathcal{E}$: unrestricted existential quantification of the form $\exists R.C$.
  $\exists$hasChild.Male

  Note: the FOL equivalent uses variables:
  $p(x) \leftrightarrow \exists y(\text{hasChild}(x, y) \wedge \text{male}(y))$),
  or $\exists$hasChild.hasChild.$\top$ for grandparents.
- $\mathcal{N}$: (unqualified) cardinalities of roles ("number restrictions").
  $(\geq 3$ hasChild.$\top)$ for persons who have at least 3 children.
- $\mathcal{Q}$: qualified role restrictions like $(\leq 2$ hasChild.Male$)$. A weaker form, $\mathcal{F}$, is restricted to cardinalites 0, 1 and "arbitrary".

The Extended Languages

- $\mathcal{AL}$ has no “branching” (no union, or any kind of disjunction; so tableau proofs in $\mathcal{AL}$ are linear.
  Exercise: show why unrestricted existential quantification $\exists R.C$ in contrast to $\exists R.\top$ leads to branching.
- The logics are named by the letters, e.g. $\mathcal{ALUN}$ for $\mathcal{AL}$ with union and unqualified n-cardinalities.
- $\mathcal{U}$ and $\mathcal{E}$ can be expressed by $\mathcal{C}$.
  Thus, $\mathcal{ALC}$ is frequently used.
- $\mathcal{ALC}$ is the “smallest” Description Logic that is closed wrt. the set operations.
- A frequently used restriction of $\mathcal{AL}$ is called $\mathcal{FL}^-$ (for “Frame-Language”), which is obtained by disallowing negation completely (i.e., having only positive knowledge).

Complexity and Decidability: Overview

- Logic $\mathcal{L}^2$, i.e., FOL with only two (reusable) variable symbols is decidable.
- Full FOL is undecidable.
- DLs: incremental, modular set of semantical notions.
- only part of FOL is required for concept reasoning.
- $\mathcal{ALC}$ can be *expressed* by FOL, but then, the inherent semantics is lost $\rightarrow$ full FOL reasoner required.
- Actually, $\mathcal{ALC}$ can be encoded in FOL by only using two variables $\rightarrow$ $\mathcal{ALC}$ is decidable.
- Consistency checking of $\mathcal{ALC}$-TBoxes and -ABoxes is PSPACE-complete (proof by reduction to *Propositional Dynamic Logic* which is in turn a special case of propositional multimodal logics).
  There are algorithms that are efficient in the average case.
- $\mathcal{ALCN}$ goes beyond $\mathcal{L}^2$ and PSPACE. Reduction to $\mathcal{C}^2$ (including “counting” quantifiers) yields decidability, but now in NEXPTIME). There are algorithms for $\mathcal{ALCN}$ and even $\mathcal{ALCQ}$ in PSPACE.

Further Extensions

- *Role Constructors*, i.e., derived roles as union or intersection (hasChild $\equiv$ hasSon $\cup$ hasDaughter), concatenation (hasGrandchild $\equiv$ hasChild $\circ$ hasChild), transitive closure (hasDescendant $\equiv$ hasChild$^+$) (indicated by e.g. $\mathcal{ALC}_{reg}$), and inverse (isChildOf $\equiv$ hasChild$^-$) ($\mathcal{I}$).
- *Data types* (indicated by "(D)"), e.g. integers.
  Adult $\equiv$ Person $\sqcap$ $\exists$age. ≥ 18.
- *Nominals* ($\mathcal{O}$) allow to use individuals from the ABox also in the TBox.
  GermanCity $\equiv$ $\forall$inCountry.Germany
  They are used in a class constructor like one-of$\{$o$_1$,...,o$_n\}$ (for defining enumeration concepts) or in has-value$\{$x$\}$ for value constraints of properties.
- Role-Value-Maps:
  Equality Role-Value-Map: $(R_1 = R_2) \equiv \{x \mid R_1(x,y) \leftrightarrow R_2(x,y)\}$.
  Containment Role-Value-Map: $(R_1 \subseteq R_2) \equiv \{x \mid R_1(x,y) \rightarrow R_2(x,y)\}$.
  knows $\subseteq$ likes for people who like all people they know.

SEMANTICS OF EXTENSIONS

- $(\geq nR.C)^{\mathcal{I}} = \{x \mid \#\{y \mid (x,y) \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\} \geq n\}$,
- $(\leq nR.C)^{\mathcal{I}} = \{x \mid \#\{y \mid (x,y) \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\} \leq n\}$,
- $(nR.C)^{\mathcal{I}} = \{x \mid \#\{y \mid (x,y) \in R^{\mathcal{I}}$ and $y \in C^{\mathcal{I}}\} = n\}$,
- $(R \circ S)^{\mathcal{I}} = \{(x,z) \mid \exists y : (x,y) \in R^{\mathcal{I}}$ and $(y,z) \in S^{\mathcal{I}}\}$,
- $(R^{-})^{\mathcal{I}} = \{(y,x) \mid (x,y) \in R^{\mathcal{I}}\}$,
- $(R^{+})^{\mathcal{I}} = (R^{\mathcal{I}})^{+}$.
- If Nominals are used, $\mathcal{I}$ also assigns an element of $\mathcal{D}$ to each nominal symbol x.
  $\{i_1, \ldots, i_n\}^{\mathcal{I}} = \{i_1^{\mathcal{I}}, \ldots, i_n^{\mathcal{I}}\}$, and
  $R.y = \{x \mid \{z \mid (x,z) \in R^{\mathcal{I}}\} = \{y\}$.

COMPLEXITY OF EXTENSIONS

- Role constructors: $\mathcal{ALC}_{reg}$, including transitivity, composition and union is EXPTIME-complete; this stays the same when inverse roles and even cardinalities for *atomic* roles are added ($\mathcal{ALCQI}_{reg}$).
  Recall that inverse and transitive closure are important for ontologies.
- Combining such *composite* roles with cardinalities becomes undecidable (encoding in FOL requires 3 variables).
- Encoding of Role-Value Maps with composite roles in FOL is undecidable (encoding in FOL requires 3 variables; the logic loses the *tree model property*).
- $\mathcal{ALCQI}_{reg}$ with role-value maps restricted to boolean compositions of *basic* roles remains decidable. Decidability is also preserved when role-value-maps are restricted to functional roles.

Description Logic Model Theory

The definition is the same as in FOL:

- an interpretation is a model of an ABox A if
  - for every atomic concept C and individual x such that $C(x) \in A$, $x^{\mathcal{I}} \in C^{\mathcal{I}}$, and
  - for every atomic role R and individuals x, y such that $R(x, y) \in A$, $(x^{\mathcal{I}}, y^{\mathcal{I}}) \in R^{\mathcal{I}}$.
- note: the interpretation of the non-atomic concepts and roles is given as before,
- all axioms ϕ of the TBox are satisfied, i.e., $\mathcal{I} \models \phi$.

Based on this, DL entailment is also defined as before:

- a set Φ of formulas entails another formula Ψ (denoted by $\Phi \models \psi$), if $\Psi^{\mathcal{I}} =$ true in all models of Φ.

Decidability, Complexity, and Algorithms

Many DLs are decidable, but in high complexity classes.

- decidability is due to the fact that often *local* properties are considered, and the verification proceeds tree-like through the graph without connections between the branches.
- This locality does not hold for cardinalities over composite roles, and for role-value maps – these lead to undecidability.
- Reasoning algorithms for $\mathcal{ALC}$ and many extensions are based on tableau algorithms, some use model checking (finite models), others use tree automata.

Three types of Algorithms

- restricted (to polynomial languages) and complete
- expressive logics with complete, worst-case ExpTime algorithms that solve realistic problems in "reasonable" time. (Fact, Racer, Pellet)
- more expressive logics with incomplete reasoning.

EXAMPLE

- Given facts: Person $\equiv$ Male $\sqcup$ Female and Person(unknownPerson).
- Query ?-Male(X) yields an empty answer
- Query ?-Female(X) yields an empty answer
- Query ?-(Male $\sqcup$ Female)(X) yields unknownPerson as an answer
- for query answering, *all* models of the TBox+ABox are considered.
- in some models, the unknownPerson is Male, in the others it is female.
- in all models it is in (Male $\sqcup$ Female).