

Rules: rules like this

$$h \leftarrow b$$

$$P(x, y, z) \leftarrow q(x, y) \wedge \exists w: r(y, z) \wedge \dots$$

allow this?  
 a how -> ?  
 => DST Return

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variable-restricted subsets of FOL:  $x=z$

$\forall x, y: \text{isUncle}(x, y) \leftarrow \exists z: \text{hasChild}(y, z) \wedge \text{sibling}(x, z)$

$\rightarrow$  need 3 variables

$\text{isUncle}(x)$  "x is an uncle of something"

$\forall x: \text{isUncle}(x) \leftarrow \exists z: (\text{sibling}(x, z) \wedge \exists x: \text{child}(z, x))$

blue x DL expression: syntactically: use only two vars: x, z

$\text{Uncle} \equiv \text{Person} \sqcap \exists \text{sibling}. (\exists \text{child}. \text{Person})$

$\text{Parent} \equiv \text{Person} \sqcap \exists \text{child}. \text{Person}$

$\text{Uncle} \equiv \text{Person} \sqcap \exists \text{sibling}. \text{Parent}$

$\text{Adult} \equiv \text{Person} \sqcap \exists \text{age} \geq 18$

$\text{Bla} = \text{Male} \sqcap \text{Female}$

$\rightarrow$  Bla must be empty!  
 "inconsistent class"

The diagram shows three variables: x, y, and z. A green arrow labeled 'sibling' points from x to z. Another green arrow labeled 'child' points from z to x. A red arrow labeled 'child' points from y to z.

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Slide 52 Example:

$LL \sqsubseteq \text{Italian}$   
 $LL \sqsubseteq \text{Person}$   
 $LL \sqsubseteq \text{Gentleman}$   
 $LL \sqsubseteq \text{English}$

Ontology:  
 $\text{Italian} \sqcap \text{English} \equiv \emptyset$

$\models LL \sqsubseteq \text{English} \sqcap \text{Italian}$   
 $\models LL \sqsubseteq \emptyset$   
 $\Rightarrow LL$  is inconsistent

from "coercion":  
 $\text{Italian} \sqsubseteq L_{923} \sqcup LL$   
 $\models \text{Italian} \sqsubseteq L_{924}$

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modulo ring

e.g.  $\mathbb{Z}_5$

$= \{ 0, 1, 2, 3, 4 \}$

double of 0:

|   |     |
|---|-----|
| 0 | 0   |
| 1 | 2   |
| 2 | 4   |
| 3 | ... |
| 4 | ... |

$6 \text{ mod } 5 \rightarrow 1$   
 $8 \text{ mod } 5 \rightarrow 3$

# = 5

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prove that  $F \models G$

$\Leftrightarrow$  show that  $F \wedge \neg G$  • is inconsistent  
 • has no models  
 • is unsatisfiable  
 • is impossible

$$\begin{matrix} F \\ \neg G \end{matrix}$$

$\rightarrow$  hard slide

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F:  $\forall x ((p(x) \rightarrow q(x)) \wedge (q(x) \rightarrow r(x)))$

$\neg G$ :  $\neg \forall x (p(x) \rightarrow r(x))$   
 $\exists x (p(x) \wedge \neg r(x))$

thip  $\rightarrow$   $a$

instantiate for  $a/x$

instantiate  $x/a$

$\forall x : p(x) \rightarrow q(x)$   
 $\forall x : q(x) \rightarrow r(x)$   
 $p(a) \rightarrow q(a)$   
 $q(a) \rightarrow r(a)$   
 $r(a)$   
 $\square$

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Example modus ponens:

axiom  $\forall x (\text{rains} \rightarrow \text{umbrella}(x))$

fact rains

there is a lise

$\Rightarrow$  instantiation  
 $x \rightarrow \text{alice}$   
 $\sigma(x) = \text{alice}$

$\frac{\text{rains} \rightarrow \text{umbrella}(\text{alice}), \text{rains}}{\text{umbrella}(\text{alice})}$

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$\forall x \exists y : \text{parent}(x, y)$

$\exists y : \text{parent}(x_1, y)$

$\exists y \text{parent}(x_2, y)$

$\text{parent}(x_1, f(x_1))$

$\text{parent}(x_2, f(x_2))$

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