Se 559, Example :

$T_P^0(\emptyset) = \{ \text{facts} \ldots \}$

$T_P(T_P^0(\emptyset)) = T_P^1(\emptyset) = \{ \ldots \text{facts} \ldots \}$

$\cup \{ \text{border}(Y,X) \mid \text{border}(X,Y) \in P \}$ ↳ symmetric closure ✓

$\cup \{ \underbrace{\text{reachable}(X,Y)}_{\text{1 step "}\rightarrow\text{"}} \mid \text{border}(X,Y) \in P \}$ ≙ : neighbor

$T_P^2(\emptyset) = T_P^1(\emptyset)$

$\cup \{ \underbrace{\text{reachable}(Y,X)}_{\text{1 step "}\leftarrow\text{"}} \mid \text{border}(X,Y) \in P \}$

$\cup \{ \underbrace{\text{reachable}(X,Y)}_{\text{2 step}} \mid \exists z: (X,Z)\ \text{1-step} \in P + (Z,Y)\ \text{neighbors} \}$

⋮

$T_P^i(\emptyset) =$ symmetric closure: { border(X,Y) in both directions)

$\cup \{ \text{reachable}(X,Y) \mid \exists z: \text{border}(X,Z) \in P \wedge X\, Z \ldots Y \text{ in } i \text{ steps} \}$ i=4 (a→d, f→e→p)

⋮

$\cup \{ \text{reachable}(X,Y) \mid \text{in } i-1 \text{ steps} \}$

fixpoint : $T_P^i(\emptyset) = T_P^{i-1}(\emptyset) = \mathcal{M}$, the minimal model

$\Longrightarrow$ tc completely computed

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note : is every country reachable from itself ?

e.g. border(a,d) ⇝ border(d,a)

⇝ reachable(a,a)

But reachable(is) ? % Iceland

no! since no borders at all.

↳ example little bit dirty

Clean up:

- add rule reachable(X,X) :- true. ↶ not safe

  ... there is no borders of Island → does not exist in P

- ↳ add atoms country(d), country(a), country(is), ...

  and then reachable(X,X) :- country(X).

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consider to add an atom to $\mathcal{M}$:

$\mathcal{M}_1 := \mathcal{M} \cup \{ reachable(is, m) \}$ % iceland, malta

$\mathcal{M}_1 \models P$

, but adds an invented "not supported" fact.

$\mathcal{M}_2 := \mathcal{M} \cup \{ reachable(m, i) \}$ % italy, malta

$\mathcal{M}_2 \not\models P$ ← because 3. rule requires

$reachable(i,a) \leftarrow reachable(m,i), border(i,a)$

⇒ when inventing/adding a fact, its consequences must also be added

$\Rightarrow \mathcal{M}_2' = \mathcal{M}_2 \cup \{ reachable(m,x) \mid reachable(i,x) \}$

note: $\mathcal{M}_2' \not\ni reachable(a,m)$ since the (i,m)→direction is not in $\mathcal{M}_2$

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Sl. 563

Resolution calculus

· we have RULES $\quad h \Leftarrow b$

↑ conjunctive formula

$\subsetneq$

Datalog: $h \text{ :- } b_1, b_2, b_3.$

FOL: $h \Leftarrow b_1 \wedge b_2 \wedge b_3.$

take something (some calculus) from FOL reasoning

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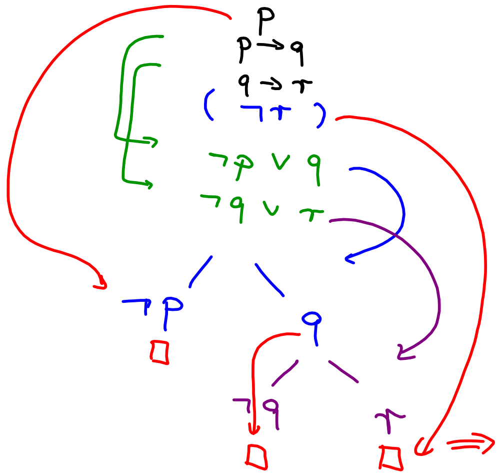
Consider FOL / Boolean logic Tableau calculus first

(recall: no explicit quantifiers, only rules)

$P = \{$
$q \leftarrow p,$
$r \leftarrow q,$
$p \}$

query: $?- r$

Tableau:



"forward reasoning"

⇒ answer: r must hold!

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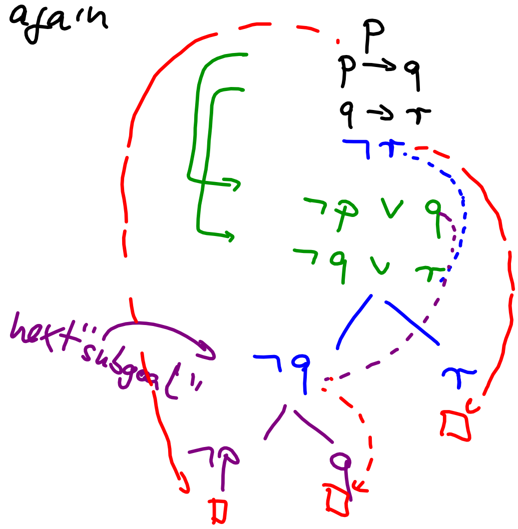
Same again



← first "goal"

"Backward"

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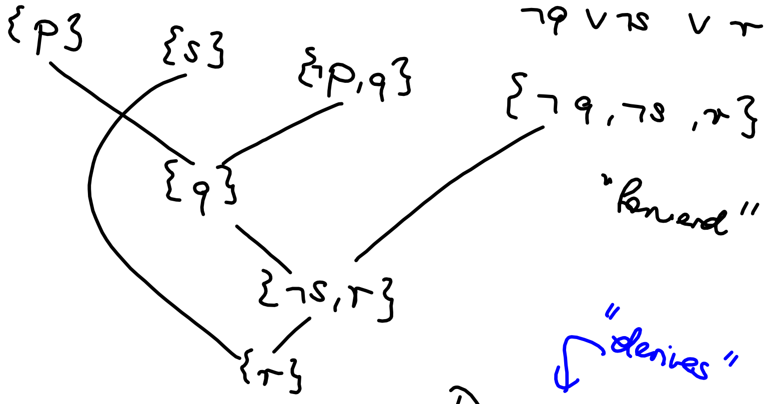
Resolution calculus

$p, \; s, \; p \to q, \; q \wedge s \to r$

$\neg p \vee q \qquad \neg(q \wedge s) \vee r$

$\neg q \vee \neg s \vee r$



"forward" $\Big\} \downarrow T_P^{\omega}$

"derives"

$P \vdash_{\text{resolution}} r$

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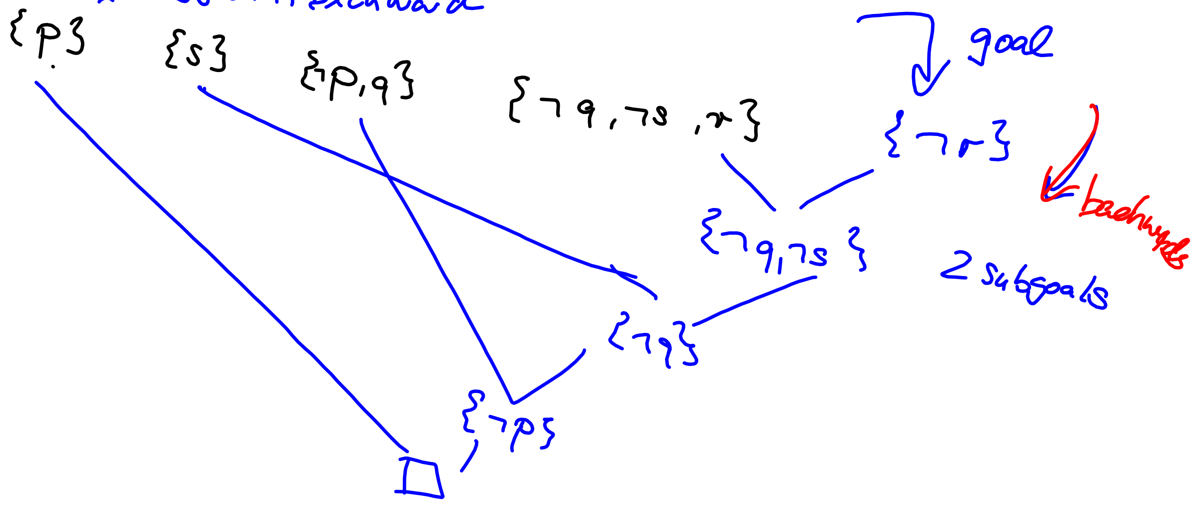
same again, backward



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