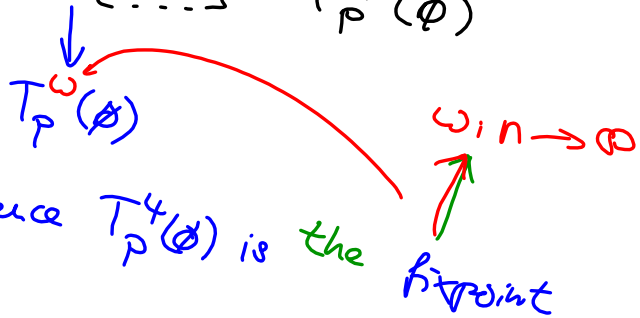


SESS1: example program (the first one) P

$$T_P^0(\emptyset) \dots T_P^5(\emptyset) = \{ \dots \} = T_P^4(\emptyset)$$



here: finite sequence (Datalog)

$T_P^4(\emptyset)$ is the

fixpoint

May 18-14:12

Considers another example with a prohibition symbol:

(Prolog)

$P = \{ \text{person}(\text{joe}), \text{person}(\text{father}(X)) :- \text{person}(X). \}$

Signature of P: $\text{person}/P, \text{father}/P, P$
 Herbrand-universe for Σ : $\{ \text{joe}, \text{father}(\text{joe}), \text{father}(\text{father}(\text{joe})) \dots \}$
 → is infinite

$$T_P^0(\emptyset) = \{ \text{person}(\text{joe}) \}$$

$$T_P^1(\emptyset) = \{ \text{person}(\text{joe}), \text{person}(\text{father}(\text{joe})) \}$$

$$T_P^2(\emptyset) = \{ \text{person}(\text{joe}), \text{person}(\text{father}(\text{joe})), \text{person}(\text{father}(\text{father}(\text{joe}))) \}$$

$$T_P^h(\emptyset) = \{ \text{person}(\text{father}^i(\text{joe})) \mid i = 0 \dots h \}$$

finite
no fixpoint

but an infinite fixpoint: (but finitely describable)

$$\{ \text{person}(\text{father}^n(\text{joe})) \mid n \in \mathbb{N} \} =: \mathcal{F}$$

d.h. $T_P(\mathcal{F}) = \mathcal{F}$

May 18-14:28

"Minimal Model"

program P from slide 55-1

- For each P , $T_P^0(\emptyset) \models P$

$T_P^0(\emptyset) = \{p, q, r, s\} \models P$ (set of atomic facts)

\mathcal{I} is a Herbrand interpretation

$\mathcal{I} \models P$ (all knowledge concluded from P)

Let $P' := P \cup \{t \leftarrow \neg u\}$

SH: $T_{P'}^0(\emptyset) = \mathcal{I}$ (the new rule has no consequences)

$\mathcal{I} \models P'$

Claim: $\mathcal{I} \cup \{t\} \models P'$ ✓
 $\mathcal{I} \cup \{u\} \not\models P'$? ...
 $\mathcal{I} \cup \{u\} \models P$ ✓ (P does not forbid u , it has no statement about u)
 $\mathcal{I} \cup \{u\} \not\models P'$ (P: if u then \perp)
 $\mathcal{I} \cup \{t, u\} \models P$
 $\mathcal{I} \cup \{t, u\} \models P'$

\Rightarrow take a model \mathcal{M} interpret something $\mathcal{M} \cup \{...\}$ extended and add necessary consequences \rightarrow new model \mathcal{M}

facts not supported by P , but also not forbidden.

\Rightarrow every (positive) P has many models

\Rightarrow but there is a **unique** MINIMAL model = $T_P^0(\emptyset)$ which satisfies only the facts that are forced by P .

"NEGATION BY DEFAULT"

May 18-14:42

Consider negation:

$P' := P \cup \{t \leftarrow \neg u\}$

$T_{P'}^0(\emptyset) = \{p, t\}$

\vdots

$T_{P'}^1(\emptyset) = \{p, q, r, s, t\}$

$P'' := P \cup \{t \leftarrow \neg r\}$

$T_{P''}^0(\emptyset) = \{p, t\}$

$T_{P''}^1(\emptyset) = \{p, q, t\}$

$T_{P''}^2(\emptyset) = \{p, q, r, t\}$

$T_{P''}^3(\emptyset) = \{p, q, r, s\}$

$= T_{P''}^4(\emptyset)$

$t \leftarrow \neg r$ does not derive anything more!

note (slide 550)

$T_{P''}^4(\emptyset) = \bigcup_{i=0}^4 = \{p, q, r, s, t\}$

we have $T_{P''}^4(\emptyset) \models P''$ (looks useless...)

but not the minimal model!

\rightarrow next slide;

May 18-14:46

Consider again

$$P'' = P \cup \{t \leftarrow \neg r\}$$

We have, $\{p, q, r, s, t\} \models_{\text{FOL}} P''$

$\{p, q, r, s\} \not\models_{\text{FOL}} P''$ the universal model

May 18-15:11