SE551: example program (the first one) P

$T_P^0(\emptyset) \ldots \; T_P^5(\emptyset) = \{\ldots\} = T_P^4(\emptyset)$

$T_P^{\omega}(\emptyset)$

$\omega;\ n \to \infty$

here: finite sequence $T_P^4(\emptyset)$ is the fixpoint

(Datalog)

May 18-14:12

Consider another example with a function symbol: (Prolog)

Signature Σ of P: person/P,1 (joe/F,0) father/F,1

Herbrand-universe for Σ:

$\{$joe, father(joe), father(father(joe)), ...$\}$ is infinite

$P = \{$ person(joe).,

person(father(X)) :- person(X). $\}$

$T_P^0(\emptyset) = \{$person(joe)$\}$

$T_P^1(\emptyset) = \{$person(joe), person(father(joe))$\}$

$T_P^2(\emptyset) = \{$person(joe), person(father(joe)), person(father(father(joe)))$\}$

$\vdots$

$T_P^n(\emptyset) = \{$person(father^i(joe)) $\mid i = 0 \ldots n\}$

no finite fixpoint

but an infinite fixpoint: (but finitely describable)

$\{$person(father^n(joe)) $\mid n \in \mathbb{N}\} =: \mathcal{F}$

d.h. $T_P(\mathcal{F}) = \mathcal{F}$

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"Minimal Model"

program P from Slide 551,

- For each P, $T_P^\omega(\emptyset) \models_{FOL} P$ — P is a set of FOL rules / FOL formulas

$T_P^\omega(\emptyset) = \{p, q, r, s\} \models P$ $=: \mathcal{H}$ is a Herbrand interpretation = set of atoms/facts — only for positive rules

"all knowledge concluded from P"

Let $P' := P \cup \{ t \leftarrow u \}$

Still. $T_{P'}^\omega(\emptyset) = \mathcal{H}$ (the new rule has no consequences)

$\mathcal{H} \models P'$

Claim: $\mathcal{H} \cup \{t\} \models_{FOL} P'$ ✓

$\mathcal{H} \cup \{u\} \models_{FOL} P'$? ...

$\mathcal{H} \cup \{u\} \models_{FOL} P$ ✓ (P does not forbid u, it makes no statement about u)

$\mathcal{H} \cup \{u\} \not\models_{FOL} P'$ — P': if u then t !

$\mathcal{H} \cup \{t, u\} \models P$

$\mathcal{H} \cup \{t, u\} \models P'$

⇒ take a model $\mathcal{M}$, invent something $\mathcal{M} \cup \{\ldots\}$ extended and add necessary consequences → new Model $\mathcal{M}'$

facts not supported by P, but also not forbidden.

⇒ every (positive) P has many models

⇒ but there is a unique MINIMAL model $= T_P^\omega(\emptyset)$ which satisfies only the facts that are forced by P.

"NEGATION BY DEFAULT"

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Consider negation:

$P' := P \cup \{ t \leftarrow \neg u \}$ (from Slide 551)

$T_{P'}^0(\emptyset) = \{p, t\}$

$\vdots$

$T_{P'}^4(\emptyset) = \{p, q, r, s, t\}$

$P'' := P \cup \{ t \leftarrow \neg r \}$

$T_{P''}^0(\emptyset) = \{p, t\}$

$T_{P''}^1(\emptyset) = \{p, q, t\}$

$T_{P''}^2(\emptyset) = \{p, q, r, t\}$

$T_{P''}^3(\emptyset) = \{p, q, r, s\}$

$= T_P^4(\emptyset)$

$t \leftarrow \neg r$ does not derive anything more!

note (Slide 550)

$T_{P''}^\omega(\emptyset) = \bigcup_{i=0..n} = \{p, q, r, s, t\}$ looks useless...

we have $T_{P''}^\omega(\emptyset) \models P''$,

but not the minimal model!

⇝ next slide:

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Consider again

$$P'' = P \cup \{t \leftarrow \neg r\}$$

We have, $\{p, q, r, s, t\} \models_{FOL} P''$

$\{p, q, r, s\} \models_{FOL} P''$ the minimal model

May 18-15:11