Ex. 1

Consider $\forall x : \boxed{F(x)}$

show: equivalent to $\neg \exists y : \neg F(y)$

consider arbitrary $\mathfrak{I} = (I, \mathfrak{D})$

$\mathfrak{I} \models_{\emptyset} \forall x : F(x)$

$\Leftrightarrow$ for all $d \in \mathfrak{D}$, $\mathfrak{I} \models_{\{x \mapsto d\}} F(x)$

$\Leftrightarrow$ math. Argumentation [same as $\emptyset^d_x$]

there is no $d \in \mathfrak{D}$ s.t. $\mathfrak{I} \not\models_{\{x \mapsto d\}} F(x)$

$\Leftrightarrow$ there is no $d \in \mathfrak{D}$ s.t. $\mathfrak{I} \models_{\{x \mapsto d\}} \neg F(x)$

$\Leftrightarrow$ not (there is some $\mathfrak{D}$ s.t. .....) back to FOL

$\Leftrightarrow$

not $\mathfrak{I} \models \exists x : \neg F(x)$

not language → math symbol about logics

$\Leftrightarrow$ $\mathfrak{I} \not\models \exists x : \neg F(x)$

and it into logical language

$\Leftrightarrow$ $\mathfrak{I} \models \neg \exists x : \neg F(x)$

rename Variable

$\Leftrightarrow$ $\mathfrak{I} \models \neg \exists y : \neg F(y)$

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Ex 1, b) "implication"

humans: think causally

if ... then .....

logic: material (= not necessarily causal) implication

$\varphi \to \psi$

whenever $\varphi$ holds ⇝ then $\psi$ holds (in an interpretation)

Proof of "causal implies material"

Consider only structures where whenever $\mathfrak{D}$ is satisfied then also $\psi$ must be satisfied

$\mathfrak{I} \models \varphi \to \psi$

$\Leftrightarrow$ $\mathfrak{I} \models \neg\varphi \vee \psi$

$\Leftrightarrow$ $\mathfrak{I} \models \neg\varphi$ or $\mathfrak{I} \models \psi$

two cases:

if $\mathfrak{I} \models \neg\varphi$ then "yes"

if not $\mathfrak{I} \not\models \neg\varphi$, i.e. then $\mathfrak{I} \models \varphi$

then since we have a causal implication IN REALITY

we must have $\mathfrak{I} \models \psi$ ⇒ yes.

$\Rightarrow$ in all cases $\mathfrak{I} \models \varphi \to \psi$

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c) Examples where $\varphi \rightarrow \psi$ is satisfied, although

- not causal ✓
- or even silly.

Signature for mondial, $\mathcal{J}$ is the mondial DB

$\mathcal{J} \models$ encompassed("Germany", "Europe") → yes

→ borders("RA", "BR") → yes

$\mathcal{J} \models \overbrace{\text{encompassed}(4, \text{"North Sea"})}^{F} \rightarrow \overbrace{\text{borders}(\text{"BR"}, \text{"RA"})}^{T}$

$\mathcal{J} \models \underbrace{\text{encompassed}(4, \text{"NorthSea"})}_{F} \rightarrow \underbrace{\text{borders}(\text{"Berlin"}, \overset{V}{\text{"45.012"}})}_{F}$ ✓

F → everything

no statement about anything

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Use of → in constraints:

$\mathcal{J} \models \forall X, Y: (\text{encompassed}(X, Y) \rightarrow \text{country}(X) \wedge \text{continent}(Y))$

⇝ state Axioms, specification

Spec often of the form

$\varphi = (F_1 \wedge F_2 \wedge \dots \wedge F_3) \longrightarrow (G_1 \wedge G_2 \wedge G_3)$

then consider only statements of the form

$\mathcal{J} \overset{?}{\models} \varphi \rightarrow (\forall x \dots)$

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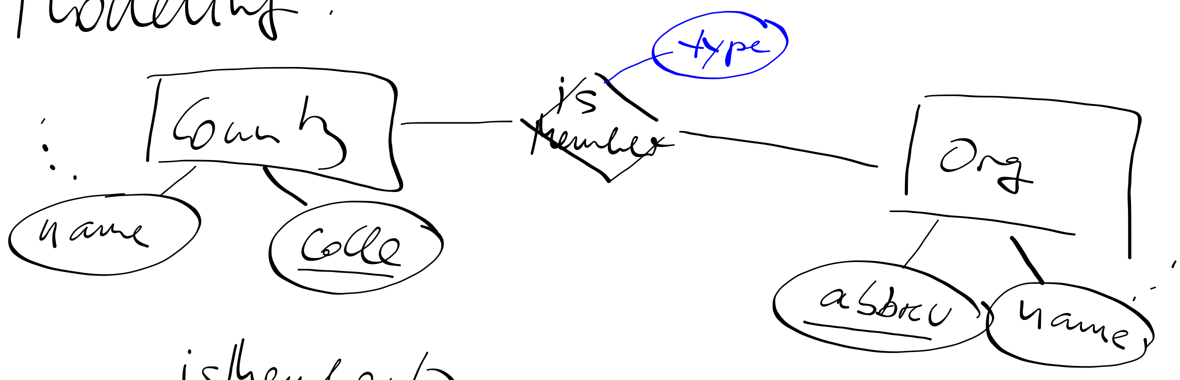
# Modeling:




isMember/2 cannot express this

note: isMember/3, e.g. (D, EU, "full Member")

workaround: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not safe!

$$F'(X) \equiv \neg S(X, b)$$

answer substitutions:

obviously $\beta_1 = \{X \mapsto 2\}$

$\beta_2 = \{X \mapsto 3\}$ active domain (1)

but also $\beta_3 = \{X \mapsto 4\}$

$\beta_{1000000} = \{X \mapsto 1\,000\,000\}$

$\beta_{alice} = \{X \mapsto alice\}$

$\vdots$

$\infty$

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