FOL:

	Terms	Formulas
Syntax	c, X, $f(a)$, $g(f(a), b)$, alice	$t_1 = t_2$, $p(t_1, t_2, \dots t_k)$, $\neg F$, $F \vee G$, $F \wedge G$, $\exists x: F(x)$, $\forall x F(x)$ ⋮
Semantics	a thing of the world (domain) (+ interpretation)	Formula can be true or false w.r.t. to a given world (interpretation)

$\mathcal{D} = \{Alice, John, \dots\}$

father(alice)

$I(alice) \longrightarrow Alice$

$I(father): Alice \mapsto John$

$[\![alice]\!]_I = Alice$

$[\![father(alice)]\!] = (I(father))(I(alice))$

$= (I(father))(Alice) = John$

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Consider $\exists$, $\forall$

$\exists x : p(x)$ $\rightsquigarrow$ $p(x)$ holds for some value that have to be considered

$\forall x : p(x)$ $\rightsquigarrow$ for all ... for X

$\forall x : p(1)$ ... holds if $p(1)$ holds

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relational Interpretation e.g.

$\mathcal{D} = \{ Alice, John, \ldots \}$

$I(alice) = Alice$
$I(john) = John$

hasFather is encoded into a predicate hasFather/2

$I(hasFather) = \{ (alice, john), (\cdot, \cdot), \ldots \}$

$[\![hasFather(alice, john)]\!]$? alice $\mapsto$ Alice, john $\mapsto$ John · $(Alice, John) \in I(hasFather)$ ⇝ yes

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better examples later ⇒ Slide 4M

$\exists x\, \forall y : (p(x,y)) \rightarrow \forall x\, \exists y : p(x,y)$

equiv

$\exists x\, \forall y : (p(x,y)) \rightarrow \forall u\, \exists w : p(u,w)$

⇒ This formula holds in any structure !

"tautology"

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4.11 "Erfüllbare" Example

$\exists x, y: \; p(x) \wedge q(x, y)$

$\mathcal{D} = \{1, 2\}$

$I(p) = \{(1)\}$

$I(q) = \{(1, 2)\}$

$\mathcal{D} = \{1\}$

$I(p) = \{(1)\}$

$I(q) = \{(1, 1)\}$

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