Ex. 1 d)

fr = ∅ free = {x}

fr = {y} free = {x,y}

fr = ∅

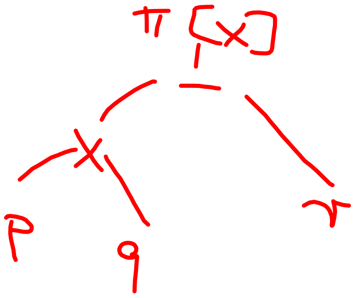
fr = {y} fr = {x,y}

$$F(x) = p(x) \wedge \exists y\,(q(y) \wedge \neg r(x,y))$$

customer(x) ∧ ∃y (product(y) ∧ ¬ordered(x,y))

$$\Rightarrow \cancel{p(x) \wedge} \exists y\,(p(x) \wedge q(y) \wedge \neg r(x,y))$$

redundant



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1 e: ¬∃

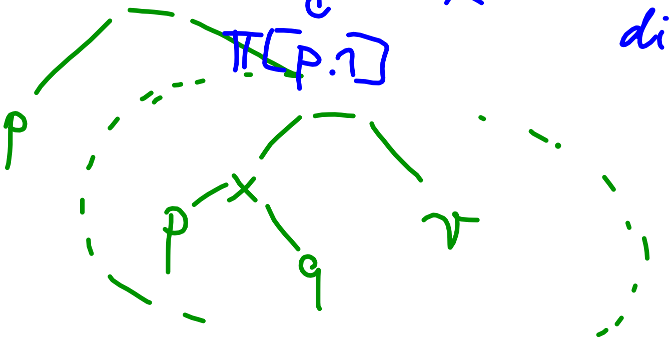
≙ relational division pattern

$$p(x) \wedge \neg\exists y\,(q(y) \wedge \neg r(x,y))$$

⇒ "minus"

intuitive way: as always done for the relational division

the x



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1(?)

free $= \{x, y, v\}$ ↯

rr $= \{v, x\}$

free $= \{y, x, w\}$ ↯

rr $= \{w, y\}$

$$\underbrace{\exists v\big((r(v,x) \land \neg s(x,y,v))\big)}_{F_1} \land \quad \leftarrow \text{push-into-exists}$$

$$\underbrace{\exists w\,\underbrace{(r(w,y) \land \neg s(y,x,w))}_{G}}$$

rr $= \{x, y\}$

free $= \{x, y\}$ ... all ... safe

but not RANF

push into ... rights, do,

$$F_1 \land \exists w (F_1 \land G)$$

$$F_1 \land \exists w \big(\underbrace{\exists v\big((r(v,x) \land \neg s(x,y,v))\big)}_{F_1} \land (r(w,y) \land \neg s(y,x,w))\big)$$

free $= \{v, x, y\}$

rr $= \{v, x\}$ ↯

$$\equiv F_1 \land \exists w \exists v \big(r(v,x) \land \neg s(x,y,v) \land r(w,y) \land \neg s(y,x,w)\big)$$

free $= \{v, x, y, w\}$

rr $= \{v, x, y, w\}$

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instead: reformulate formula at the beginning:

$F(x,y)$

$$\exists v\big((r(v,x) \land \neg s(x,y,v))\big) \land \exists w\,(r(w,y) \land \neg s(y,x,w))$$

$$\equiv$$

$$\exists v,w : \big(r(v,x) \land r(w,y) \land \neg s(x,y,v) \land \neg s(y,x,w)\big)$$

rr $= \{v, x, w, y\}$ ✓

free $= \{v, x, w, y\}$

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another possibility with push:

$$\exists v\big((r(v,x) \land \neg s(x,y,v)) \land \exists w\,(r(w,y) \land \neg s(y,x,w))\big)$$

$\pi T = \{w, y\}$

free $= \{y, \cancel{x}, w\}$

problem: only for x

→ need pos. binding for x

$$F_1 \land \exists w\,(r(v,x) \land r(w,y) \land \neg s(y,x,w))$$

$\pi T = \{v, x, w, y\}$

free $= \{v, x, w, y\}$ ✓

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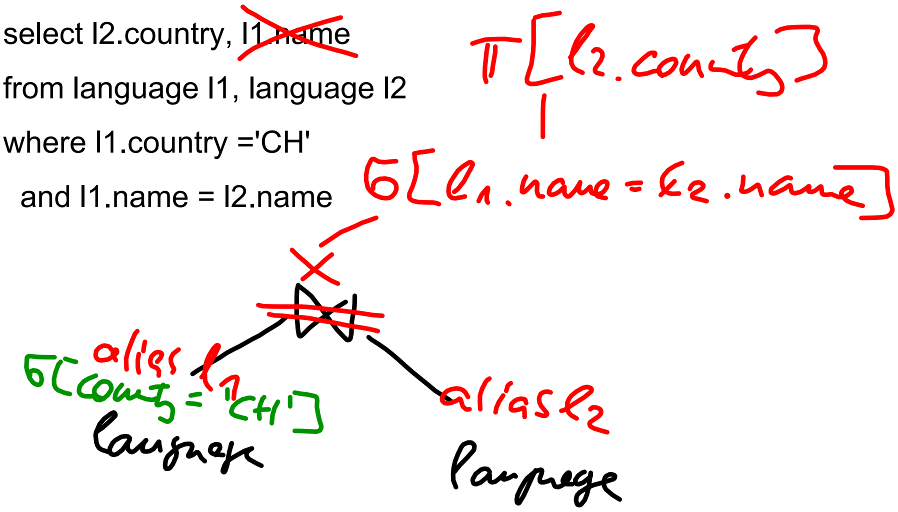
Ex. 2a)

select l2.country, ~~l1.name~~
from language l1, language l2
where l1.country ='CH'
and l1.name = l2.name



$$F(C) = \exists\, L, P_1, P_2 :$$
$$language('CH', L, P_1) \land language(C, L, P_2)$$

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2b)

$$F(C) \equiv \exists N, P, A, Gp, P : \text{Country}(C, N, P, A, Gp, CP)$$

$$\wedge\ \forall \text{ languages in that country}$$

$$\forall L\ (\exists P_2 : \text{language}(C, L, P_2)$$

$$\rightarrow \text{ not spoken in CH}$$

$$\neg \exists P_1 : \text{language}(\text{'CH'}, L, P_1))$$

$\forall \rightarrow \neg\exists$

$$\equiv \exists N, P, A, Gp, P : \text{Country}(C, N, P, A, Gp, CP)$$

$$\wedge\ \neg\exists L\ (\exists P_2, P_1 : \text{language}(C, L, P_2) \wedge \text{language}(\text{'CH'}, L, P_1))$$

Ex 2 a

$\pi[\text{code}]$ Country — 2a

(select code from country)
MINUS
(select l2.country
from language l1, language l2
where l1.country ='CH'
and l1.name = l2.name)

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c)

$$F(C) \equiv \exists N, P, A, Gp, CP : \text{country}(C, N, P, A, Gp, CP)$$

$$\wedge\ \forall L\ (\exists P_1\ \text{language}(C, L, P_1) \rightarrow \exists P_2\ \text{language}(\text{'CH'}, L, P_2))$$

$$\equiv \exists N, P, A, Gp, CP : \text{country}(C, N, P, A, Gp, CP)$$

$$\wedge\ \neg\exists L\ (\exists P_1\ \text{language}(C, L, P_1) \wedge \neg\exists P_2 : \text{language}(\text{'CH'}, L, P_2))$$

≙ all countries except those where a non-swiss lg. is spoken

NonSwissLgCountry

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2c SQL:

```
(select code from country)
MINUS
(select l1.country
 from language  l1
 where l1.name not in (select name from language
           where country='CH'))
```

$\hat{=}$ $\neg\exists$

Swiss languages

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d)

$$F(C) \equiv \exists N, P, A, Cap, CP \; \mathrm{Country}(C, N, P, A, Cap, CP)$$

$$\land \; \forall lg \text{ in 'CH'} \rightarrow lg \text{ spoken in } C$$

$$\forall L: (\exists P_1: \mathrm{language}('CH', L, P_1) \rightarrow \exists P_2: \mathrm{language}(C, L, P_2))$$

$$\hat{=} \; \exists \ldots \mathrm{country}(C, \ldots)$$

$$\land \neg\exists L\,(\exists P_1: \mathrm{language}('CH', L, P_1) \land \neg\exists P_2: \mathrm{language}(C, L, P_2))$$

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