Exercise 3, Sheet 2:

Program P from Slide 572 (borders, reachable)

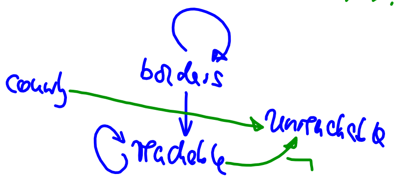
$P =: \{r_1, r_2, r_3\}$

$P \cup \{$ unreachable(X,Y) :- country(X), country(Y), not reachable(X,Y). $\}$

$=: r_4$

$\Rightarrow$ stratify it:

dep graph:



S_1 : EDB relations

· everything that depends only positive (+ recursive processing of reach.):

$S_1 = \{$country, borders, reachable$\}$

$S_2 = \{$unreachable$\}$

associated stratification of the program:

$P_1 = \{r_1, r_2, r_3\}$

$P_2 = \{r_4\}$

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... evaluate it:

$J_0 := \emptyset$

$J_1 := \cancel{J_0} \cup T^{\omega}_{P_1}(J_0)$

$= \{$ borders(a,b) : a,b is a country and a and b are neighbors $\}$

$\cup \{$reachable(a,b): b is reachable from a $=$ tc(borders)$\}$

(cf. induction proof from last week)

$\cup \{$all facts from mondial$\}$

$J_2 := J_1 \cup T^{\omega}_{P_2}(J_1)$

$= J_1 \cup \{$unreachable(a,b) s.t. $(a,b) \notin$ tc(borders)$\}$

important! contains all things derived before!

$= \mathcal{S}(P)$ is the unique "stratified model"

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.. back to the exercise sheet:

- Stratifiable P may have multiple minimal models:
  - $\leadsto$ give just some example case
- for P, $\varphi(P)$ is a minimal model ✓:
- some additional minimal model:
  - "insert" border(de, aus):
  - $\mathcal{H} := \varphi(P) \cup \{\text{borders}(de, aus)\}$
    - it is not a model!
    - for a model, also
      - border(aus, de) must be there
      - readable(d, aus) "
      - reachable(ch, aus) "
  - consider rule r4: ("unreachable")
  - goes into $\mathcal{I}_1'(P) \supsetneq \mathcal{I}_1(P)$
  - "non-minimal model of P_1"
  - $P_2 = \{r4\}$ does not any more derive unreachable(de, aus) ...
  - get $\varphi'(P) := \mathcal{I}_2'$
  - now we have $\varphi'(P)$ is a model,
  - and $\varphi'(P) = (\varphi(P) \cup \{\dots\}) \setminus \{\dots\}$
  - i.e. not a proper superset of $\varphi(P)$!
  - is it minimal?
    - (can we remove something and we still have a model?)
  - ... yes, e.g. remove {border(de, aus), border(aus, d...
    - then it is still a model
    - if remove reachable(de, aus)
      - $\hookrightarrow$ needs unreachable(de, aus) ↯
  - next page

$\varphi'(P) - \{\ \} =: \mathcal{M}_1$

$\mathcal{M}_1$ is a model.

$\mathcal{N}_1 := \mathcal{M}_1 - \{\ \}$ is not a model,

but $\mathcal{N}_1 \cup \{\text{unreachable}(de, \dots$

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Consider now the nonmonotonicity:

$$\underbrace{\varphi(P \cup \text{Mondial})}_{\text{input}} \hat{=} \varphi(P) \text{ before}$$

$$\supseteq \{\text{unreachable}(de, aus)\}$$

$$\varphi(\underbrace{P \cup \text{Mondial} \cup \{\text{border}(de, aus)\}}_{\text{more input}}) \not\supseteq \{\text{unreachable}(de, aus)\}$$

some of the before conclusions is not valid any more

$\Rightarrow$ closed world reasoning is nonmonotonic

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Use small examples for such things:

Boolean

$p :- \neg q$

FOL

$p(a) :- q(a)$

Dep. Graph: $p \xleftarrow{-} q$

$S_1 = \{q\}$
$S_2 = \{p\}$

$P_1 = \emptyset$
$P_2 = \{p :- \neg q\}$

$J_0 = \emptyset$
$J_1 = \emptyset \cup T_{P_1}^{\omega}(\emptyset) = \emptyset$
$J_2 = \emptyset \cup T_{P_2}^{\omega}(\emptyset) = \{p\}$

$\Rightarrow \varphi(P) = \{p\}$

- is a model
- is minimal

Consider $\mathcal{M} := \{q\}$

- is a model
- is minimal

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back to the lecture:

Why is this $\mathcal{M}$ not as good as $\varphi(P)$

- both are models
- both are minimal

?

Consider: apply P to $\mathcal{M}$ or $\varphi(P)$

$T_P^{\cancel{\omega}}(\varphi(P)) = \{p\} = \varphi(P)$

$T_P^{\cancel{\omega}}(\mathcal{M}) = \emptyset$

does not reproduce the "knowledge"

$\Rightarrow \mathcal{M}$ was invented, not "supported".

ω is not necessary to check this

$\Rightarrow T_P^{\omega}$ to construct $\varphi(P)$ is polynomial (no. of steps)

Check is only linear :)

but: to check, we have to guess :(

(actually exponential)

$\Rightarrow$ idea towards STABLE MODELS

guess and check ....

(for stability)

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Idea for checking stability from before:

- good (see example)
- not perfect:

bad example:

$$P = \{ p \leftarrow p \}$$

even without negation....

original definition $T_P^{\omega}(\emptyset) = \emptyset$ is the minimal model

our above check $T_P(\{p\}) = \{p\}$ survives this check :-(

→ linear check is not that good.....

but a good start....

see later....

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