- Rules with negation:
  - bottom-up T_P^ω and Stratification (✓)
  - model-theoretic characterization ... later
  - proof-theoretic characterization
    - T_P^ω finite only for Datalog, not for Prolog
    - Prolog (XSB) based on the proof-theoretic char.

Jun 5-14:06

resolution proof idea:

?- res(X, Y, Z).

for a program P that defines res/3 in its rules res(X,Y,Z)

$\{\neg res(X,Y,Z)\}$ $\qquad$ $\{res(X,Y,Z), \neg \ldots, \neg \ldots\}$

σ $\qquad$ σ

$\{\neg \ldots, \neg \ldots\}$

σ

$\square$

denote answer bindings for X,Y,Z from σ

and only if

$\hat{=}$ if we can close the resolution proof for some X/a, Y/b, Z/c then we have shown that $P \vdash res(a,b,c)$

$\hat{=}$ when we cannot close the proof, ... then we can $\boxed{\text{not}}$ derive res(a,b,c)

$\hat{=}$ if the proof fails, then we say $P \nvdash res(a,b,c)$

interpret this as $P \vdash \neg res(a,b,c)$

"Negation as failure"

(= failure to prove...)

$\hat{=}$ Closed-World negation

$\Rightarrow$ Reasoning mechanism for Closed-World negation

Jun 5-14:25

Slide 553 (borders, reachable, finite fragment of border instances)

$T_P^0(\emptyset) = \emptyset$

border("A", "D")

$T_P^1(\emptyset) = T_P(\emptyset) = \{$ border(a, d), ... border(bol, br) $\}$

only the facts, all given neighbourships in one direction

$T_P^2(\emptyset) = T_P(T_P^1(\emptyset)) = \{$

new from Rule 1: b(d, a) ...... b(br, bol), inverse ~~symm~~

Rule 2: r(a, d) .... r(bol, br)

1-step reachability for the original instances of borders (non-symmetric!)

border is now symmetric

Rule 3: nothing

instances of facts: b(a, d), ... b(bol, br) $\}$

Lemma 1: in $T_P^2(\emptyset)$, border is symmetric.

reachable: 1-step, if step is in the "base"-direction

$T_P^3(\emptyset) = T_P(T_P^2(\emptyset)) = T_P(\{ \; \ldots\ldots \; \})$

$= \{$

Rule 1: symm. border from symm. border

Lemma: 'border' is symmetric for all $T_P^i(\emptyset)$ s.t. $i \geq 2$

Rule 2: r(X, Y) s.t. b(X, Y) $\in T_P^{i-1}$

Lemma: 1-step reachability is complete in $T_P^i(\emptyset)$ s.t. $i \geq 3$.

Rule 3: r(a, f) — r(a, d) b(d, f). r(a, ch) — r(a, d), b(ch, d) $\leftarrow$ b(d, ch) ......

Lemma: 2-step reachability where first step is in original direction

$\}$

observation:

$$T_P^i(\emptyset) \supseteq T_P^{i-1}(\emptyset)$$

recall slide 547:

$$T_P^\omega = \bigcup_{n \in \mathbb{N}} T_P^n(\emptyset)$$

$$= \lim_{n \to \infty} T_P^n(\emptyset)$$

after a finite number of steps ....

Jun 5-14:54

Consider for our example

$T_P^{\cancel{17}\, k}(\emptyset)$

Lemma by induction

- since $T_P^2(\emptyset)$: borders is symmetric
- k-2-reachability is complete
- (k-1 reachability: where final step is in the base direction)

RinkDB .... longest path is finite.

e.g. $\underline{19}^{\,k}$ steps from P-E-F- ..... THA-MAL-SGP

$\Rightarrow$ after $T_P^{k+2}(\emptyset)$ "reachable" is complete

$$T_P^{19}(\emptyset) = T_P^{20}(\emptyset)$$

$$T_P^{21}(\emptyset) = ?\ T_P(T_P^{20}(\emptyset)) = T_P(T_P^{19}(\emptyset))$$

$$= T_P^{20}(\emptyset)$$

$\Rightarrow$ if it stops with

$$T_P^{k+2}(\emptyset) \Rightarrow T_P^{k+2}(\emptyset) = \lim_{n \to \infty} T_P^n(\emptyset)$$

$$= T_P^\omega(\emptyset)$$

:-)

sl 576 SAFE!

$$P' := \{\ \text{unreachable}(X,Y) :- \cancel{\text{adom}}(X), \text{adom}(Y), \text{not reachable}(X,Y)\ \}$$

country(X), country(Y)

$$T_{P'}(T_P^\omega(\emptyset))$$

$\to$ fills unreachable correctly :-)

"Schichtung"

Jun 5-14:57

$$T_{P \cup P'}(\emptyset) = \{ \text{border}(a,d), \ldots \text{border}(bd, br), \text{country}(a), \text{country}(d), \ldots \}$$

$$T_{P \cup P'}(T_{P \cup P'}(\emptyset)) = \{ \text{facts} + T_{P_1}(\emptyset) \} \cup \underbrace{\{ \text{unreachable}(a,d), \ldots \}}_{\text{unreachable} = \text{Country} \times \text{Country} \setminus \emptyset}$$

Jun 5-15:07

⇒ break the program into "strata" levels

$P_0, P_1, P_2, \ldots$

s.t. when P_i uses $\neg p(\ldots)$

then p must be completely defined in P_{i-1} (or P_{i-2} or before)

Jun 5-15:15

Base relations 'EDB' r, s, t

$r(\ldots)$

$n \leftarrow s \wedge t$

$q \leftarrow r \wedge \neg s$

$p \leftarrow t \wedge \neg q$

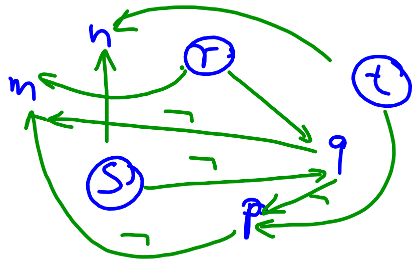
$m \leftarrow r \wedge \neg p \wedge \neg q$

level 0: r, s, t EDB

level 1: q, n

level 2: p

level 3: m



#4 ✓

level 0: s, r, t EDB

level 1: n, q

level 2: p

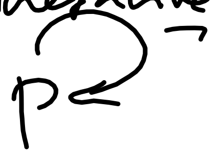
level 3: m

#4

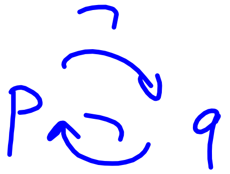
⇒ graph wrt "¬" must be acyclic

Jun 5-15:17

Shortest negative cycle:



$p(X) \text{ :- } adom(X), \neg p(X).$



$p \leftarrow \neg q$

$q \leftarrow \neg p$

⇒ not stratifiable

Our current (stratified) model theory does not talk about that!

Jun 5-15:28

Consider once more

$p \leftarrow \neg q$ is stratifiable

"if we cannot derive q, then we are willing to believe in p"

intended model: $\{p\}$ ← "supported" by Negation as Default

$\emptyset$ is not a model: $T_P(\emptyset) = \{p\}$

$\Rightarrow$ $\{p\}$ is a minimal model

$T_P(\{q\}) = \{q\}$ is a model, is minimal

but I invented it, it is not supported

$\Rightarrow$ two minimal models

$T_P(\{r\}) = \{p, r\}$ is also a model

but it is not a model

$\Rightarrow$ if we learn (invent) another fact, we can complete to a model

AS LONG AS OUR MODELS TALK ONLY ABOUT <u>POSITIVE</u> Knowledge

Jun 5-15:30

Back to

$p \leftarrow \neg q \overset{FOL}{\cong} p \vee q$

$q \leftarrow \neg p \overset{FOL}{\cong} p \vee q$

$$\begin{tikzcd} p \arrow[r, bend left, "\neg"] & q \arrow[l, bend left, "\neg"] \end{tikzcd}$$

$A \to B$

$\neg A \vee B$

FOL: $p \vee q$

"intended" models: $\{p\}$ $\{q\}$ minimal both somehow supported

$\{p, q\}$

Jun 5-15:38