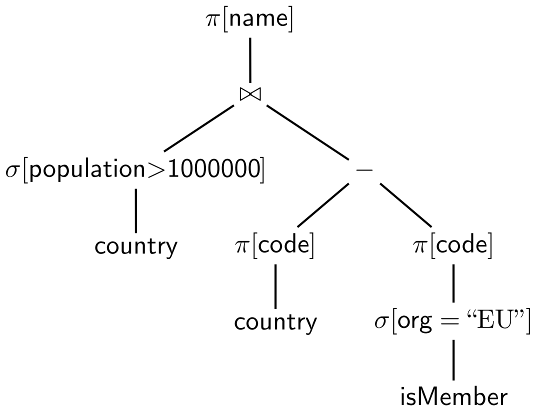
Exercise 3 (RANF to Algebra – Minus) Give expressions in the relational algebra and in the relational calculus for the query *"Full names of all countries that have more than 1000000 inhabitants and are not member of the EU"*.

Check whether the calculus expression is in SRNF and RANF, and transform it into the relational algebra. Compare the result with the algebra expression.

A straightforward algebra expression is



The calculus expression is

$$F(N) = \exists C, Cap, CapProv, A, Pop : \\ (\mathsf{country}(N, C, Cap, CapProv, A, Pop) \wedge Pop > 1000000 \wedge \neg\exists T : \mathsf{isMember}(C, \text{“EU”}, T))\ .$$

It is in SRNF, it is safe range, and it is in RANF. Recall that for the subformula $\neg\exists T : \mathsf{isMember}(C, \text{“EU”}, T)$, RANF requires $rr(\exists T : \mathsf{isMember}(C, \text{“EU”}, T)) = free(\exists T : \mathsf{isMember}(C, \text{“EU”}, T)) = \{C\}$ which is the case.

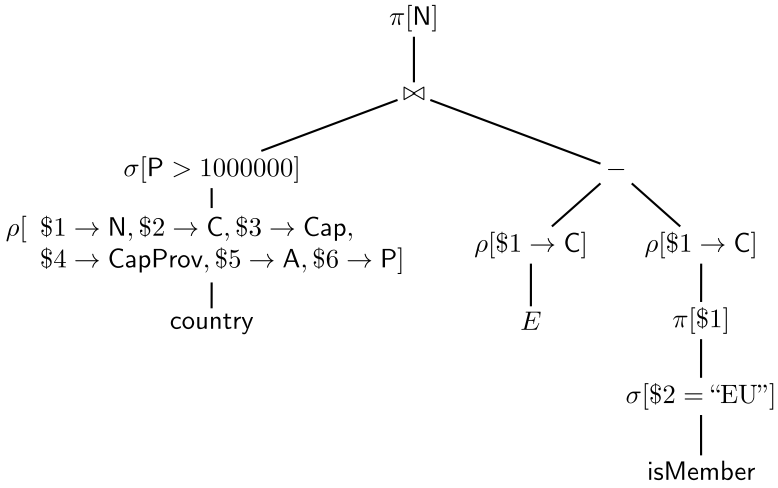
For the relational algebra,

$$\begin{array}{lcl} \mathsf{isMember}(C, \text{“EU”}, T) & \Rightarrow & \rho[\$1 \to C, \$3 \to T](\pi[\$1, \$3](\sigma[\$2 = \text{“EU”}](\mathsf{isMember}))) \\ \exists T : \mathsf{isMember}(C, \text{“EU”}, T) & \Rightarrow & \pi[\$1](\rho[\$1 \to C, \$3 \to T](\pi[\$1, \$3](\sigma[\$2 = \text{“EU”}](\mathsf{isMember})))) \\ & = & \rho[\$1 \to C](\pi[\$1](\sigma[\$2 = \text{“EU”}](\mathsf{isMember}))) \end{array}$$

For $\neg\exists T : \mathsf{isMember}(C, \text{“EU”}, T)$, let the expression E denote the algebra expression that enumerates all values of the active domain. With this,

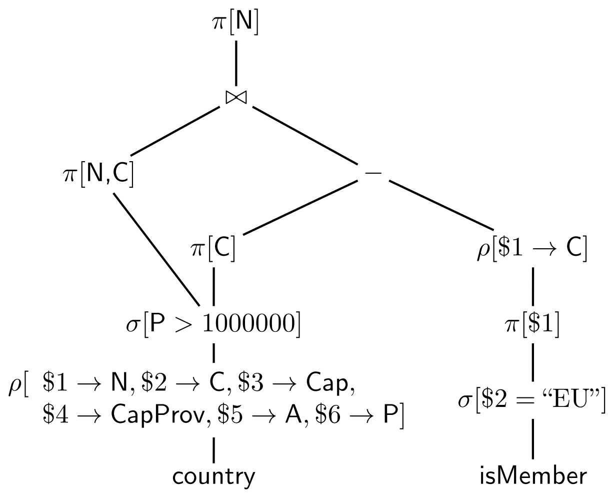
$$\neg\exists T : \mathsf{isMember}(C, \text{“EU”}, T) \Rightarrow \rho[\$1 \to C](E) - \rho[\$1 \to C](\pi[\$1](\sigma[\$2 = \text{“EU”}](\mathsf{isMember})))$$

Altogether, the whole query translates to

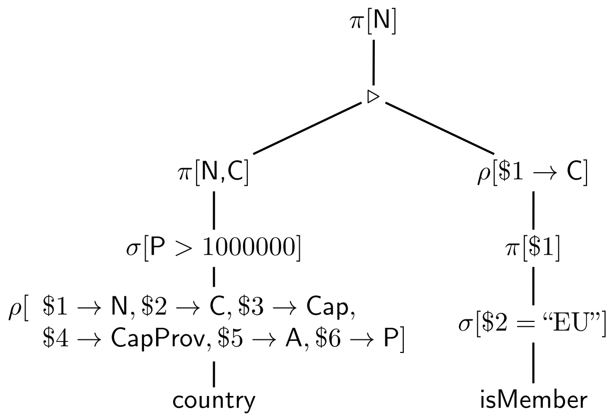


Obviously, the term $\rho[\$1 \to \mathsf{C}](E)$ can be replaced by $\rho[\$2 \to \mathsf{C}](\pi[\$2](\mathsf{country}))$ which enumerates a superset of all values of C that can result from the left subtree.

Instead, also $\rho[\$2 \to \mathsf{C}](\pi[\$2](\sigma[\$6 > 1000000](\mathsf{country})))$ is sufficient, which makes the left subtree (nearly) unnecessary. From it, only the full name must still be obtained.



Another possibility is the anti-join $\triangleright$ (which is one of the built-in operators of internal algebras):



Exercise 4 (Division: Äquivalenz von Algebra und Kalkül) For the relational algebra, the division operator has been introduced as a derived operator (cf. lecture "Databases"). Consider the relation schemata $r(A, B)$ and $s(B)$.

$$r \div s = \{\mu \in Tup(A) \mid \{\mu\} \times s \subseteq r\} = \pi[A](r) \setminus \pi[A]((\pi[A](r) \times s) \setminus r).$$

Derive a query in the relational calculus from the left-hand side, and prove the equivalence with the right-hand side.

The left-hand side expression: the set of all possible tuples over a A is described by $F(X) = ADOM(X)$. The remaining task is then easy: for all values Y in S, the combination of X and Y must be in R:

$$F(X) = ADOM(X) \wedge \forall Y : (s(Y) \rightarrow r(X, Y)) \ .$$

Here, it is obvious that instead $ADOM(X)$, the consideration can be restricted to the A-values of R:

$$F(X) = \exists Z : r(X, Z) \wedge \forall Y : (s(Y) \rightarrow r(X, Y)) \ .$$

The query is not in SRNF. It is equivalent to

$$F(X) = \exists Z : r(X, Z) \wedge \neg\exists Y : (s(Y) \wedge \neg r(X, Y)) \ ,$$

which is in SRNF (thus, domain-independent), but not in RANF.

Transformation to RANF ("push-into-not-exists"):

$$F(X) = \exists Z : r(X, Z) \wedge \neg\exists Y : (\exists Z_2 : r(X, Z_2)) \wedge s(Y) \wedge \neg r(X, Y))$$

Derivation of the algebra expression:

F	$Algebra$
$(\exists Z : r(X,Z)) \wedge s(Y) \wedge \neg r(X,Y)$	$(\pi[A](r) \times s) \setminus r$
$\exists Y : (\exists Z : r(X,Z)) \wedge s(Y) \wedge \neg r(X,Y)$	$\pi[A]((\pi[A](r) \times s) \setminus r)$
	(the expression has the format A)
$\exists Z : r(X,Z)$	$\pi[A](r)$ (has again the format A)
$F(X)$ as above	$\pi[A](r) \setminus \pi[A]((\pi[A](r) \times s) \setminus r)$

... is exactly the right-hand side.

Exercise 5 (Kalkül: Gruppierung und Aggregation) Define a syntactical extension for the relational calulus, that allows to use aggregate functions similar to the `GROUP BY` functionality of SQL.

For this, consider only aggregate functions as simple applications over single attributes like $\mathsf{max(population)}$, but not more complex expressions like $\mathsf{max(population/area)}$.

- What is the result of an aggregate function, and how can it be used in the calculus?
- Which inputs does an aggregate function have?
- how can this input be obtained from the database?

Give a calculus expression for the query "For each country give the name and the total number of people living in its cities".

The result is a number. It can be bound to a variable or it can be used in a comparison. Thus, the aggregate function is to be considered as a term (whose evaluation yields a value).

The immediate input to an aggregate function is a set/list of values, over which the aggregate is computed (sum, count, ...).

This list can be obtained as results of a (sub)formula (similar to a correlated subquery) with a free variable.

The results are grouped by zero, one or more free variables of the subquery. Usually, these also occur in other literals outside the aggregation.

$$X = \textsf{\textit{agg-op}}\{\textsf{\textit{var}}\ [\textsf{\textit{group-by-vars}}]; \textsf{\textit{subq-fml}}\}$$

where in *subq-fml* the *group-by-vars* and *var* have free occurrences. E.g.,

$$\begin{array}{l} F(CN, SumCityPop) = \\ \qquad \exists C, A, P, Cap, CapProv : country(CN, C, A, P, Cap, CapProv) \wedge \\ \qquad SumCityPop = \mathsf{sum}\{CityPop\ [C]; \\ \qquad\qquad\qquad\qquad \exists CtyN, CtyProv, L1, L2 : city(CtyN, CtyProv, C, CityPop, L1, L2)\} \end{array}$$

groups by C, computes the sum over $CityPop$ and binds the value to $SumCityPop$.

Comments:

- a similar syntax is used in F-Logic;
- the usage in XSB is similar, but the user has to program it more explicitly:
  - the list is created by the Prolog predicate "bagof";
  - the aggregation operation over the list must be programmed in the common Prolog style for handling a list.

Exercise 6 (Kalkül→Algebra) Consider the relation schemata $R(A,B), S(B,C)$ und $T(A,B,C)$.

a) Give an equivalent algebra expression for the following safe relational calculus expression:

$$F_1(X,Y) = T(Y,a,Y) \wedge (R(a,X) \vee S(X,c)) \wedge \neg T(a,X,Y)$$

Proceed as shown in the lecture for the equivalence proof.

b) Simplify the expression.

c) Extend the expression from 8a) to

$$F_2(Y) = \exists X : (F_1(X,Y) \wedge X > 3)$$

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a) First, consider each of the three conjuncts (denoted as F_2, F_1 and F_3) separately:
The literal $F_1(Y) = T(Y,a,Y)$ corresponds to the subexpression

$$E_1 = \rho[A \to Y](\pi[A](\sigma[(A = C) \wedge (B = a)](T))) \ .$$

The subformula $F_2(X) = R(a,X) \vee S(X,c)$ corresponds to the expression

$$E_2 = \rho[B \to X](\pi[B](\sigma[A = a](R))) \cup \rho[B \to X](\pi[B](\sigma[C = c](S))) \ .$$

Negated literal $F_3(X,Y) = \neg T(a,X,Y)$: The literal $F_4(X,Y) = T(a,X,Y)$ corresponds to the expression

$$E_4 = \rho[B \to X, C \to Y](\pi[B,C](\sigma[A = a](T)))$$

According to the lecture, the expression corresponding to $F_3(X,Y)$ is then

$$E_3 = \rho[\$1 \to X, \$2 \to Y](ADOM^2) - \rho[B \to X, C \to Y](\pi[B,C](\sigma[A = a](T)))$$

where $ADOM^2 = ((\pi[A](R) \cup \pi[B](R) \cup \pi[B](S) \cup \pi[C](S) \cup \pi[A](T) \cup \pi[B](T) \cup \pi[C](T)) \times (\pi[A](R) \cup \pi[B](R) \cup \pi[B](S) \cup \pi[C](S) \cup \pi[A](T) \cup \pi[B](T) \cup \pi[C](T)))$ contains all 2-tuples of values from the database.
Thus, $E = E_1 \bowtie E_2 \bowtie (ADOM^2 - E_4)$ is the complete algebra expression.

b) Simplify: E_1 and E_2 have no variable/column in common, thus it can be simplified as $(E_1 \times E_2) \bowtie (ADOM^2 - E_4)$. Both subterms bind X and Y, thus, $ADOM^2$ can be omitted, obtaining $E' = (E_1 \times E_2) - E_4$.

c) The additional comparison is expressed as a selection, and the $\exists X$ quantification is expressed as a projection to Y:

$$\pi[Y](\sigma[X > 3](E'))$$