Chapter 4
Internal Organization and Implementation

This section heavily relies on other subdisciplines of Practical Computer Science:

- System Structures, down to the physical level
- Operating Systems Aspects: Caching
- Algorithms (mainly: for joins) and Data Structures (tree indexes, hashing)

PHYSICAL DATA ORGANIZATION

- the **conceptual schema** defines which data is described and its semantics.
- the **logical schema** defines the actual relation names with their attributes (and datatypes), keys, and integrity constraints.
- the **physical schema** defines the **physical database** where the data is actually stored.
  $\Rightarrow$ efficiency
- system: the data is actually stored in **files**: data that semantically belongs together (a relation, a part of a relation (**hashing**), some relations (**cluster**)).
- additionally, there are files that contain auxiliary information (**indexes**).
- data is accessed **pagewise** or **blockwise** (typically, 4KB – 8KB).
- each page contains some **records** (tuples). Records consist of **fields** that are of an elementary type, e.g., bit, integer, real, string, or pointer.

DB Server Architecture: Secondary Storage and Caching

runtime server system: accessed by user queries/updates

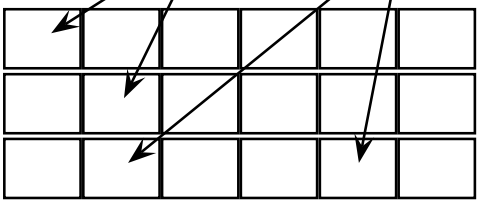
- parser: translates into algebra, determines the required relations + indexes
- file manager: determines the file/page where the requested data is stored
- buffer/cache manager: provides relevant data in the cache
- query/update processing: uses only the cache

Cache (main memory): pagewise organized

- Accessed pages are fetched into the cache
- pages are also changed in the cache
- and written to the database later ...

Secondary Storage (Harddisk): pagewise organized

- data pages with tuples
- index pages with tree indexes (see later)
- database log etc. (see later)



Database Access Mechanism

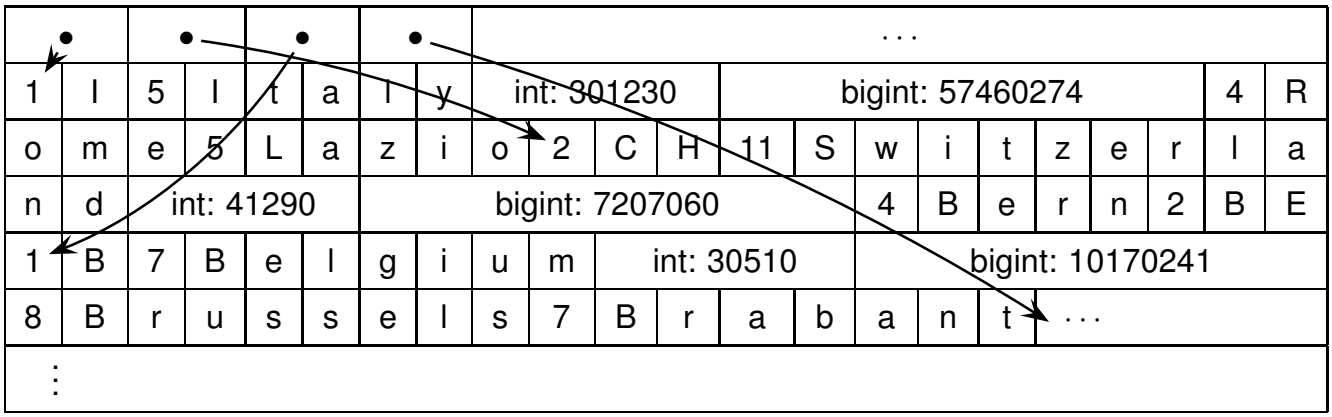
Records must be loaded from (and written to) the secondary memory for processing:

- the **file manager** determines the page where the record is stored.
- the **buffer/cache manager** is responsible to provide the page in the buffer (**buffer management**):
  - maintains a **pool** of pages (organized as frames).
    for every page, it is stored if the page has been changed, how often/frequently it has been used, and if it is currently used by transactions
  - if the required page is not in the cache, some stored page is replaced (if it has been changed, it must be written to the secondary memory)
- complex **prefetching** strategies, based on knowledge about transactions.
  [see lecture on Operating Systems]
- for now, it is sufficient to note that pagewise access has to be dealt with.

Storage of Files, Pages, and Records

- Inside a file, every tuple/record has a **tuple identifier** of the form (p, n) where p is the page number and n is its index inside the page.
  Each page then contains a **directory** that assigns a physical address to each n.
- memory management for deleted records
- different strategies for fixed-length and variable-length records

Simplified storage of a page of the Country table:



... so far to the physical facts ...

4.1 Efficient Data Access

- efficiency depends on the detailed organization and additional algorithms and data structures
- support **generic** operations:
  - **Scan:** all pages that contain records are read.
  - **Equality Search:** all records that satisfy some equality predicate are read.
    SELECT * FROM City WHERE Country = 'D';
  - **Range Search:** all records that satisfy some comparison predicate are read.
    SELECT * FROM City WHERE Population $>$ 100.000;
  - **Modify**, **Delete**: analogously
  - **Insert**: analogously: search for an appropriate place where to put the record.
- linear search (scan) ??
- Need for efficient searching (equality and/or range)

INDEXING

Indexes (for a file) are auxiliary structures that support special (non-linear) **access paths**

- Based on **search keys**
- not necessarily the relational “keys”, but *any* combination of attributes
- a relation may have several search keys
- an *index* is a set of data entries on some pages together with efficient access mechanisms for locating an entry according to its search key value.
- different types of indexes, depending on the operations to be supported:
  - equality search
  - range search (ordered values)
  - search on small domains
- in general, joins by key/foreign-key references are supported by indexes.

TREE INDEXES

This topic brings data structures and databases (= applications of data structures) together.

- introductory lecture “Computer Science I”: store numbers in trees.
- databases: tree *index* over the values of a column of a relation
  - search tree based on the values (numbers, strings)
  - the tuples themselves are not stored in the tree
  - entries (or leaf entries only) hold the values *and* point to the respective tuples in the database
- special trees with higher degrees:
  - each node (of the size of a storage page) has multiple entries and multiple children.

ASIDE, APPLICATION AND REVIEW: BINARY SEARCH TREES

- Binary Search Trees are a typical topic in “Computer Science I”: store numbers.
- Often used for *implementation* of other concepts, e.g., sets (cf. also topic “Abstract Datatypes” in some CS I lectures)
  - set: add(x), contains(x)?, list-all()
    how to implement efficiently?
    Java: classes TreeSet, HashSet

Example: BSB-based TreeSet in SQL Query Answering

[Exercise/Demonstrate on whiteboard]

(1) SELECT country FROM City WHERE population $>$ 1000000; contains duplicates.

(2) SELECT distinct country FROM City WHERE population $>$ 1000000;
  - initialize an empty TreeSet rs,
  - evaluate (1),
  - during computation, for each result r:
  - if $r \notin rs$, then add it, and output r;
  - requires $n \cdot \log n$ steps.

- sketch Java class: treenode(left: node, right: node, value: String)
- sketch representation of BSB in storage page/array (cf. CS I)
  [node at position $n \rightarrow$ left child at position $2n$, right child at position $2n+1$]

(3) SELECT country, count(*) FROM City WHERE population $>$ 1000000
GROUP BY country;
  - same strategy as above, additional data (count) stored in the tree,
  - if $r \notin rs$, then add $(r, count : 1)$ it, and output r,
  - if $r \in rs$, then increment count of r.

B- AND B*-TREES

A **B-tree** (R. Bayer & E.McCreight, 1970) of order (m, ℓ), $m \geq 3$, $\ell \geq 1$, is a search tree [see lecture on Algorithms and Data Structures]:

- the root is either a leaf or it has at least 2 children
- every inner node has at least $\lceil m/2 \rceil$ and at most m children
- all leaves are on the same level (balanced tree), and hold at most ℓ entries
- inner nodes have the form $(p_0, k_1, p_1, k_2, p_2, \ldots, k_n, p_n)$ where $\lceil m/2 \rceil - 1 \leq n \leq m - 1$ and
  - k_i are search key values, ordered by $k_i < k_j$ for $i < j$
  - p_i points to the $i + 1$th child
  - all search key values in the left (i.e., p_{i-1}) subtree are less than the value of k_i (and all values in the right subtree are greater or equal)

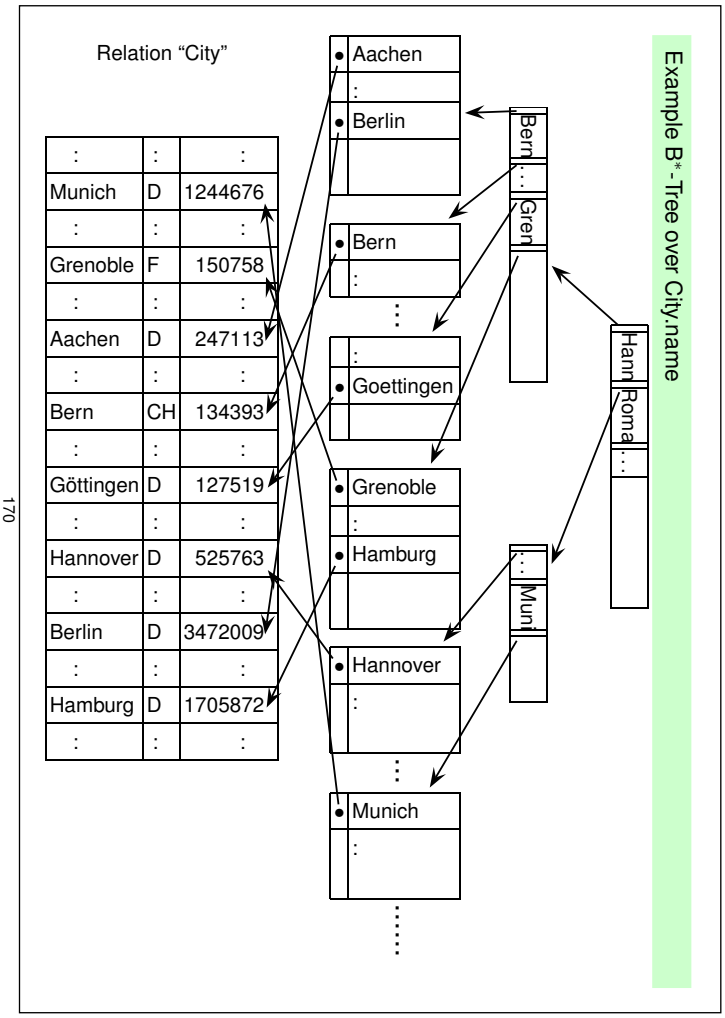
B-trees are used for "simply" organizing items of an ordered set (e.g. for sorting) as an extension of binary search trees.

B*-TREES

(sometimes also called B^+-Trees)

A **B*-tree** of order (m, ℓ), $m \geq 3$, $\ell \geq 1$, is closely related, except:

- they are intended to associate *data* with search key values:
- the inner nodes do not hold additional data, but are still intended to guide the search. (organized internally e.g. as binary search trees)
- The leaves are of the form $([k_1, s_1], [k_2, s_2], \ldots, [k_g, s_g])$ where $g \leq \ell$, k_i is a search key value, and s_i is a data record or (in databases) a pointer to the corresponding record.



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Properties

- Let N the number of entries. Then, for the height h of the tree,
  $h \leq \lceil log_{m/2}(2N/\ell) \rceil$ ($\ell/2$ entries per leaf, inner nodes half filled) and
  $h \geq \lceil log_m(N/\ell) \rceil$ (ℓ entries/leaf, inner nodes completely filled)
- equality search needs h steps
  inside each of the inner nodes, search is also in $O(n)$
- if the leaves are connected by pointers, ordered sequential access (range search) is also supported
- insertions and modifications may be expensive (tree reorganization)

Use of B*-Trees as Access Paths in Databases

- databases: cities stored in data files, index trees hold *pointers* to city records in their tree entries.
- separation between index files/pages and data files/pages.
- multiple search trees for each relation possible.

Example

Example 4.1
Consider the MONDIAL *database with 3000 cities, with an index over the name. Assume the following sizes:*

- *every leaf (tuple) page contains 10 cities,*
- *every inner node contains 20 pointers*

Then

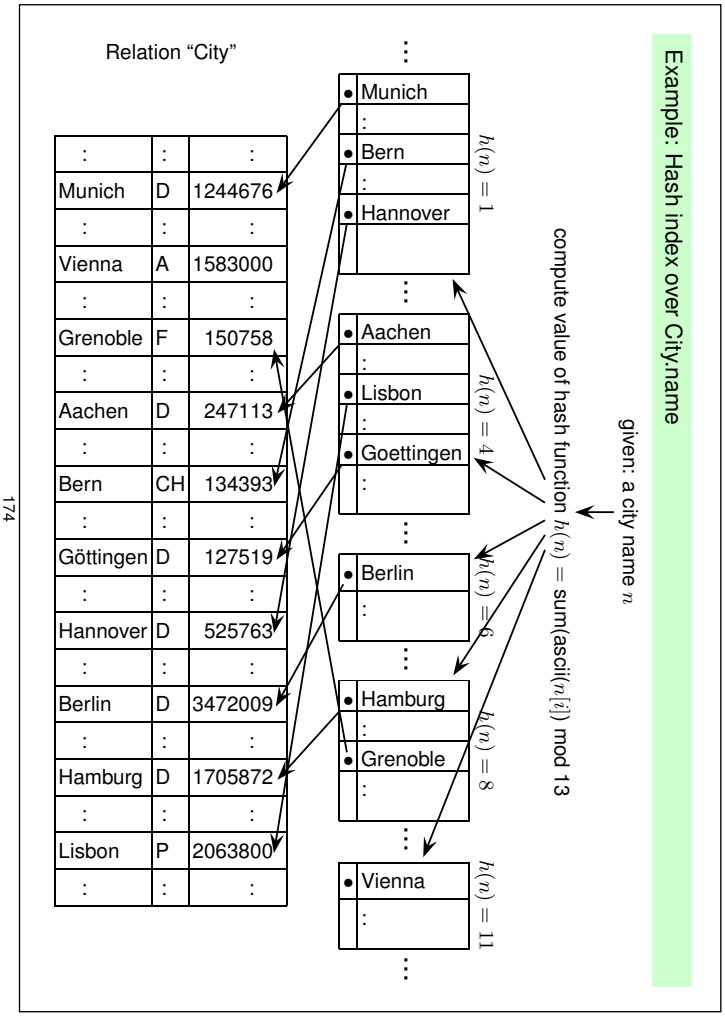
- *every inner node on the lowest level covers 200 cities*
- *every inner node on the second lowest level covers 4000 cities*
- *minimal: only one level of inner nodes*
- *maximal: two levels of inner nodes (nodes about only 2/3 filled)*
- *access every city with WHERE Name ="..." in 3 or 4 steps*
- *index on population, e.g., for WHERE Population $>$ 1.000.000 ORDER BY Population*
- *realistic numbers: block size 4K: lowest level (keys+pointers to DB): 100 cities; inner nodes: 100 references.* □

HASH-INDEX (DICTIONARY)

Hash index over the value of one or more columns ("hask key" – is not necessarily a key of the relation):

The values are distributed over k **tiles**.

- A **hash function** h is a function that maps each value to a tile number.
  operation: lookup(value)
- each tile holds pairs (key,pointer) to all tuples whose hash-key value is mapped to this tile;
- each tile consists of one or more pages.
- common technique: convert value to an integer i. i mod k gives the tile number.



Hash-Index

Example 4.2

Multi-attribute hash keys:

Consider a hash index on City.(name, province, country). h computes the sum of the ASCII numbers of the letters and takes the remainder modulo 111. □

Properties:

- equality search and insert in constant time (+ time for searching in the tile)
- does not support range queries or ordered output

Comments:

- maintenance of overflow pages: see Info III
- lookup inside each tile can be organized by a B-tree (kept on a single page)

TREE AND HASH INDEXES: OBSERVATIONS

- structure of tree leaf nodes and hash tiles is the same:
  - pairs: (search key value, reference to a tuple in storage page)
- tree leaf nodes and hash tiles are actually *projections* of a table R on the search key attribute/attributes
  - $\pi[\text{name}](\text{City})$ can be answered by just using the above tree/hash indexes.

$\Rightarrow$ indexes not only be as access paths, but also to support frequently used projections.

DUPLICATES IN TREE OR HASH INDEXES

What if $\pi[\text{search-key-attrs}](\text{relation})$ contains duplicates,
e.g., index on City.country?

- straightforward: multiple subsequent (value, ref) entries for the same value in the leaf/tile pages,
- saves some space: pairs (value, set-of-references).
- any index that is not over a superset of a candidate key potentially contains duplicates.

BIT LISTS

If the number of possible values of a search key value is small wrt. the number of tuples, **bit lists** are useful as indexes

Let a an attribute of the records stored in a file f, where k values of a exist. Then, a bit list index to f consists of k bit vectors $B(v_i)$, $i \leq 1 \leq k$.

- $B(v_i)(j) = 1$ if the jth record in f has the value v_i for the attribute a.

Properties:

- access all tuples with a given value:
  without bit list: linear search over all pages
  with bit list: access bit list, and access those pages where a “1”-tuple is located.
- modifications: no problem
- deletions: depends (if gaps are allowed on the pages)

Example 4.3

Consider the relation isMember(organization, country, type) where type has only the values "member", "applicant", and "observer":

isMember		
Org.	***Country***	***Type***
EU	*D*	*member*
UN	*D*	*member*
UN	*CH*	*observer*
UN	*R*	*member*
EU	*PL*	*applicant*
EU	*CZ*	*applicant*
:	*:*	*:*

Bit list for type=member: *1 1 0 1 0 0 . . .*
Bit list for type=observer: *0 0 1 0 0 0 . . .*
Bit list for type=applicant: *0 0 0 0 1 1 . . .*

Bit list for org=EU: *1 0 0 0 1 1 . . .*
Bit list for org=UN: *0 1 1 1 0 0 . . .*

- *Search for members of the UN:*
  without bit list: linear search over all pages
  with bit list: access bit list, and access those pages where a 1-tuple is located.
- *2nd bit list on organization column: "members of UN" by logical "and" of bit lists.* □

ORDERED STORAGE

Tuples of a relation can be stored grouped/ordered wrt. one search key

- Example: table City grouped by country (or even by (country, province))

Relation "City"

Vienna	A	1583000	Innsbruck	A	118000	..	Bern	CH	134393	..	Berlin	D	3472009	Hamburg	D	1705872
Munich	D	1244676	Hannover	D	525763	Göttingen	D	127519	Aachen	D	247113	..	Paris	F	2152423	
Grenoble	F	150758	..	..	..	Lisbon	P	2063800	..	..	..					

- things that are frequently needed together can be fetched within the same page,
- Index on City.country needs only a reference to the first tuple in each country
  (note that the index is still useful to have access in $O(\log n)$ or $O(1)$).

- insertions and modifications more costly
  (strategies to allow and keep free space between blocks)

CLUSTERING

- keep data that semantically belongs together on the same pages
- obviously done for relations, by ordered storage even inside a relation
- data of several relations $R_1(\bar{X}_1), \ldots, R_n(\bar{X}_n)$ can also be grouped to **clusters**:
  - choose a set $\bar{Y} \subseteq \bar{X}_1 \cap \ldots \cap \bar{X}_n$ as **cluster key**.
  - combine relations by their $\bar{Y}$-values.
  - provides obvious advantages when evaluating joins over the cluster key.

Example 4.4

Consider the relation schemata

organization(name,abbrev,established,. . .)

and

isMember(organization, country, type).

The foreign key

isMember.organization → organization.abbreviation

is used as cluster key.

Organization	
Abbrev	

EU	***Name***	***established***
	European Union	*07.02.1992*
	Country	***Type***
	D	*member*
	A	*member*
	:	*:*

UN	***Name***	***established***
	United Nations	*26.06.1945*
	Country	***Type***
	D	*member*
	CH	*observer*
	:	*:*

4.1.1 Algorithms and Data Structures: Basic Techniques

Iteration: all tuples in the input relations are processed iteratively. If possible, instead of the tuples themselves, an index can be used.

Indexes: selections and joins can be based on processing of an index for determining the tuples that satisfy the selection condition (a join condition is also a selection condition).

Indexes on single attributes: obvious.

Indexes on multiple attributes:

- a **hash index** to a conjunction of **equality** predicates of the form **field = value** can be used if every field of the search key occurs exactly in one predicate together with a constant,
- a **tree index** to a conjunction of **comparison** predicates of the form **field** θ **value** can be used if a prefix of the search key exists such that each field of the prefix occurs in exactly one predicate together with a constant.

  Example: if (Country,Province,CityName) is the search key of an index, it can also be used as an index for the key's prefix (Country,Province).

4.1.2 Implementation of Algebra Operations

SELECTION

Selection: $\sigma[R.field\,\theta\,value]R$.

- no index, unordered tuples: linear scan of the file
- no index, tuples ordered wrt. $field$: find the tuple(s) that satisfy "$field\ \theta\ value$" (binary search) and process these tuples.
- tree index: find the tuple(s) that satisfy "$field\ \theta\ value$" using the tree index and process these tuples.
- hash: suitable only if θ is equality.

Selection: boolean combination of predicates

Boolean combinations of predicates can be evaluated by set operations on references (from the tree leaves and hash tiles):

Consider π[name](σ[country="D" $\wedge$ population > 500.000](City))

- hash index on City.country,
- tree index on City.population (supports range search for > 500.000)

Leaf entries and entries on hash tiles are of the form

(search-key-value, reference-to-tuple)

(i) hash lookup on City.country="D" results in a set of references,

(ii) index search on City.population>500.000 results in a set of references.

- compute intersection:
- put (i) into a TreeSet or HashSet s (fits in main memory)
- for each reference in (ii): if it is contained in s, access actual tuple and output its name.

PROJECTION

Projection: $\pi[field_1, field_2, ..., field_m]R$.

Main problem: remove duplicates (relational algebra does not allow duplicates, SQL does).

- if an index over $field_1, field_2, ..., field_m$ or a superset of it exists: scan only the index leaf nodes and apply a projection to them.
- by sorting:
  scan the file, create a new file with projected tuples.
  sort the new file (over all fields, $n \log n$).
  scan the result, remove duplicates.
- by hash: scan the file, put the projected tuples into a hash.
  duplicates all end up in the same tile (still in files).
  remove duplicates separately for each tile (iterating the hashing process with different functions until tile fits in main memory)
  collect the tiles in the output file.

  $\Rightarrow$ hashing not only as an index for search *structures*, but also as an *algorithm* for partitioning.

JOIN

Consider an Equijoin: $R \bowtie_{R.A=S.B} S$.

Nested-loop-join

`foreach` tuple $r \in R$ `do`

  `foreach` tuple $s \in S$ `do`

    `if` $r_A = s_B$ `then` add $[r, s]$ to result

Very inefficient:

- assumption: only one page of each relation fits into main memory
- m tuples of R per page, n tuples of S per page:
  $|R|/m + |R| \cdot |S|/n$ filesystem accesses (for each tuple of R, loop over all pages of S).
- $|R| \cdot |S|$ comparisons.

obvious: if possible, process the smaller relation in inner loop to keep it in main memory ($|R|/m + |S|/n$ filesystem accesses).

Block-nested-loop-Join

Optimization by page-oriented strategy:

Divide S into blocks that fit in main memory and process each of the blocks separately.

- assumption: only one page of each relation fits into main memory
- m tuples of R per page, n tuples of S per page:
  - take first page (R_1) of R and first page (S_1) of S;
  - combine *each* tuple of R_1 with each tuple with S_1.
  - continue with R_1 and second page (S_2) of S ... up to R_1 with S_{last}
  - continue with R_2 and S_1 etc.
- $|R|/m + |R|/m \cdot |S|/n$ filesystem accesses (for each page of R, loop over all *pages* of S).
- still $|R| \cdot |S|$ comparisons.
- same for $k > 1$ available cache pages: read $k - 1$ pages from R for each block.

Straightforward optimizations for Block-Nested-Loop-Join

For each block, some optimizations can be applied:
(that motivate also the subsequent algorithms)

- idea: do not consider cartesian product + selection as the base for join, but look up matching tuples in S.

Applied to the Block-Nested-Loop-Join base algorithm, these do not reduce the number of page accesses, but the number of comparisons:

- generate a temporary index on $S.B$ over the loaded fraction of S;
- in case of duplicates of $R.A$: generate a temporary index also over $R.A$. on the single loaded page to process tuples groupwise;
- [do not discuss yet: generate temporary ordered indexes both on $R.A$ and $S.B$, process them merge-style]

next step: consider these optimizations on the global level of the algorithm

Index-Nested-Loop-Join

If for one of the input relations there is an index over the join attribute, choose it as the inner relation.

- given index on S's join attribute
- loop over R, for each value access matching tuple(s) in S.
- $|R|/m + |Result|$ filesystem accesses,
- up to $|R|/m + |R| \cdot |S|$ filesystem accesses,
- usually less efficient than block-nested-loop!

Parallelization

- Divide R into partitions and process each of them on a separate processor (share the index on S).

Sort-merge-join

Algorithm type: “Scan line”

First: simple case that illustrates the principle:

Assumption: relations are sorted wrt. $R.A$ and $S.B$

- search for matches as follows:
  - proceed through the ordered R and S stepwise, always doing a step in the index where the “smaller” value is.
- if a match is found:
  - generate a result tuple.
  - check for tuples which have the same values of the join attributes (must immediately follow this match in both relations).
- $|R|/m + |S|/n$ filesystem accesses.

General Case: relations are not sorted

... next slide

Sort-merge-join (cont’d)

- Sort relations first:
  - sort R according to $R.A$:
  - copy first k pages in cache, sort them in place (Quicksort), store them,
  - do this for pages $2k...3k - 1$ etc.
  - requires $2 \cdot |R|/m$ page accesses (read and write)
  - mergesort the sorted packets (linear; traverse all packets in parallel and write out in different place)
  - requires again $2 \cdot |R|/m$ page accesses (read and write)
  - do the same for S

  Sort+merge: $4 \cdot |R|/m + 4 \cdot |S|/n + |R|/m + |S|/n$ page accesses
- [note: the step “mergesort the sorted packets” can be omitted by running merge join directly on all of them; then $2 \cdot |R|/m + 2 \cdot |S|/n + |R|/m + |S|/n$.]

Sort-merge-join (cont'd)

- Use only sorted indexes:
  - compute sorted indexes of both relations on the join attributes
    (fit into main memory, requires $|R|/m + |S|/n$)
    (if tree indexes on join attributes are available, simply use them)
  - search for matches as above:
    - proceed through the ordered indexes R and S stepwise, always doing a step in the index where the "smaller" value is.
  - if a match is found:
    - access actual tuples and generate a result tuple.
    - check for tuples which have the same values of the join attributes (must immediately follow this match in both relations).
  - at most $|R|/m + |S|/n + |Result|$ filesystem accesses
    (when processing duplicates with same $R.A = S.B$ value, less than $+ |Result|$)
- with sorted indexes: costs depend on number of results ("selectivity of the join").

$\Rightarrow$ decide to sort relations or to use indexes based on selectivity/heuristics.

Sort-merge-join: basic Algorithm

`if` R not sorted on attribute A, sort it;
`if` S not sorted on attribute B, sort it;
$Tr :=$ first tuple in R;
$Ts :=$ first tuple in S;
$Gs :=$ first tuple in S;
`while` $Tr \neq eof$ `and` $Ts \neq eof$ `do` {
  `while` $Tr \neq eof$ `and` $Tr.A < Ts.B$ `do` Tr = next tuple in R after Tr;
  `while` $Ts \neq eof$ `and` $Tr.A > Ts.B$ `do` $Ts =$ next tuple in S after Ts;
  // now, $Tr.A = Ts.B$: match found
  `while` $Tr \neq eof$ `and` $Tr.A = Ts.B$ `do` {
    $Gs := Ts$;
    `while` $Gs \neq eof$ `and` $Gs.B = Tr.A$ `do` {
      add $[Tr, Gs]$ to result;
      $Gs =$ next tuple in S after Gs;}
    $Tr =$ next tuple in R after Tr;
    }
  $Ts := Gs$;
}

Hash-join

Algorithm type: “Divide and Conquer”

Partitioning (building) phase:

- Partition the smaller relation R by a hash function h_1 applied to $R.A$.
- Partition the larger relation S by the same hash function applied to $S.B$.
  (into different hash tables)

Matching (probing) phase:

- potential(!) matches have been mapped to “face-to-face” partitions.
- thus, consider each pair of corresponding partitions:
- if partition of R does not fit into main memory, proceed recursively with another h.
- otherwise – i.e., R-partition fits in main memory:
  - if corresponding S-partition also fits into main memory, compute the join.
  - otherwise proceed recursively with another (finer) h_2 inside main memory and process S-partition pagewise.

EXERCISE

Exercise 4.1
Consider the join of two relations R and S wrt. the join condition $R.A = S.B$. R uses M pages with p_R tuples each, and S uses N pages with p_S tuples each. Let $M = 1000$, $p_R = 100$, $N = 500$, $p_S = 80$; the cache can keep 100 pages. Compute the number of required I/O-operations for computing the join (without writing the result relation) for:

(a) simple nested-loop-join,

(b) pagewise nested-loop-join,

(c) index-nested-loop-join (Index on S_B),

(d) sort-merge-join (sorted relation/sorted index),

(e) hash-join. □

Another Example

In the previous comparison, the index-nested-loop-join did not perform well:

- access to each individual matching tuple is fast, but every page of S has to be accessed n times.

Example 4.5

Consider $(\sigma[cond](R)) \bowtie_{R.A=S.B} S$ where $cond$ has a selectivity q, (e.g. $q = 1/1000$):

- *loop over R, evaluate $cond$, for each tuple that satisfies the condition, access matching tuple(s) in S using the index.*
- *$|R|/m + |Result| = |R|/m + q \cdot |R| \cdot q'$ filesystem accesses,*
- *where q' is the selectivity of the join.*

$\Rightarrow$ *usage of index is efficient when only a small number of tuples of S has to be accessed (the other algorithms had to process S completely).* □

SEMIJOINS

- Natural Semijoins or Condition Semijoins as subqueries:
  $R \ltimes$ [condition($R_{i_1}, ..., R_{i_1}, S_{i_1} ..., S_{i_\ell}$)]$S$
  It is often sufficient to use an index over $S_{i_1} ... S_{i_\ell}$ instead of the whole relation.
- Semjoins as preparation for joins: $R \bowtie S = (R \ltimes S) \bowtie S$ (and symmetrically $= R \bowtie (R \ltimes S)$)
  Is that simpler? – Sometimes Yes!
  - cf. Join $R \bowtie S$ with if an existing index over the join attribute(s) in S: iterate over R and access S via index. No access, if not found.
    This actually *is* $(R \ltimes S) \bowtie S$.

  Consider $R \bowtie S$ to be a hash join:
  - First hash S, build an index over the join attribute(s), keep the index in main memory.
  - iterate over R: for every tuple μ, if $\mu \in (R \ltimes S)$ (can be checked against the main memory index), then hash it.

GROUPING AND AGGREGATION

General Structure:

SELECT $A_1, \ldots, A_n$	list of attributes
FROM $R_1, \ldots, R_m$	list of relations
WHERE F	condition(s)
GROUP BY $B_1, \ldots, B_k$	list of grouping attributes
HAVING G	condition on groups
ORDER BY H	sort order

- SUM, AVG, COUNT: linear scan necessary (need to consider all tuples)
- MIN, MAX: index can be used
- support for grouping (SQL: GROUP BY) using an index or a hash table

UNION, DIFFERENCE, INTERSECTION $\Leftrightarrow$ CARTESIAN PRODUCT

- intersection and cartesian product are special cases of join
- union and difference: analogous to sort-merge-join or hash-join

SITUATION-DEPENDENT OPTIMIZATION

Determining the optimal **execution plan** depends on **cost models**, **heuristics**, and the actual physical schema and the actual database contents:

- index structures
- statistics on data (i.e., to some extent state dependent)
  - cardinality of relations
  - distribution of values

Notions

The **selectivity** sel of operations can be used for estimating the size of the result relation:

- Selection with condition p:

$$sel_p = \frac{|\sigma[p](R)|}{|R|}$$

  Proportion of tuples that satisfy the selection condition.
  - for p an equality test $R.A = c$ where A is a key of R, $sel_p = 1/|R|$.
  - if $R.A$ distributes evenly on i different values, $sel_p = 1/i$.

- Join of R and S:

$$sel_{RS} = \frac{|R \bowtie S|}{|R \times S|} = \frac{|R \bowtie S|}{|R| \cdot |S|}$$

  is the proportion of result tuples wrt. the cartesian product.

  for $R \bowtie_{R.A=S.B} S$ with A a key attribute, $|R \bowtie_{R.A=S.B} S| \leq |S|$, thus $sel_{RS} \leq 1/|R|$.

- optimal order of applying joins
- optimal order of applying selections

Application-Level Algorithmic Optimization

And never forget about using efficient algorithms for querying the database!

- analyze the problem from the algorithmic point of view
- before hacking

Examples

Use the MONDIAL database for the following examples.

Example 4.6
Compute all pairs of european countries that are adjacent to the same set of seas. □

Example 4.7
Compute all political organizations that have at least one member country on every continent (this operation is called relational division*).* □

4.1.3 Exercises and Examples

Solution to Exercise 4.1

R: 1000 pages, each of them with 100 tuples
S: 500 pages, each of them with 80 tuples
Cache: 100 pages

a) simple nested loop join

- outer loop: R
  Load each page of R. For all tuples, iterate over S's pages.
  $1000 \cdot (1$ (load) $+ 100$ (tuples per page) $\cdot 500$ (pages of $S)) = 50,001,000$
- outer loop: S
  Load each page of S. For all tuples, iterate over R's pages.
  $500 \cdot (1$ (load) $+ 80$ (tuples per page) $\cdot 1000$ (pages of $R)) = 40,000,500$

Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples
S: 500 pages, each of them with 80 tuples
Cache: 100 pages

b) pagewise nested loop:

- outer loop: R. Load each page of R. Combine all tuples of that page with all tuples from each page of S.
  $1000 \cdot (1 + 500$ (pages of $S)) = 501,000$
- outer loop: S. $500 \cdot (1 + 1000$ (pages of $R)) = 505,000$

b2) maximum-pages nested loop: load as many (first 99) pages of R as possible in the cache and join with one page of S after the other. Then continue with 2nd 99 pages.

- 11 times, i.e., $10 \cdot 99$ R pages + $1 \cdot 10$ R pages through all S pages.
  Overall: $1000 + 11 \cdot 500 = 6500$.
- symmetric with $(5 \cdot 99 + 1 \cdot 5)$ pages of S:
  Overall: $500 + 6 \cdot 1000 = 6500$.
- can be algorithmically optimized by on-the-fly indexes during the main-memory join

Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples
S: 500 pages, each of them with 80 tuples; Cache: 100 pages

c) Index-nested-loop (index on $S.B$): Assumptions:

- every $R.A$ matches – average – 4 S-tuples.
- index over $S.B$ already exists (if not: creating an index over $S.B$ requires 500 accesses) and can be kept in main memory

Iterate over R, for each tuple search for the value of $R.A$ in the index over $S.B$ and access the tuples
$1000 \cdot (1$ (load) $+ 100 \cdot 4$ (access tuples)$) = 401,000$.

Note: the number of page accesses depends on the number of results since for every actual result there is one page access.

In case that S is ordered wrt. $S.B$: $1000 \cdot (1$ (load) $+ 100 \cdot 1$ (access tuples)$) = 101,000$.

Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples
S: 500 pages, each of them with 80 tuples; Cache: 100 pages

d) sort-merge:
different settings

- if already ordered: linear scan:
  $1000 + 500 = 1500$
- sort relations first:
  $2 \cdot 1000 + 2 \cdot 500 + 1000 + 500 = 4500$
- using an index (assume: every $R.A$ matches – average – 4 S-tuples):
  $1000 + 500 + 4 \cdot 1000 \cdot 100 = 401,500$

Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples
S: 500 pages, each of them with 80 tuples
Cache: 100 pages

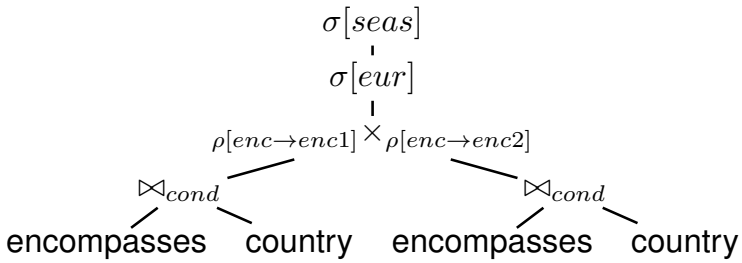
e) Hash-Join:

- Hash R:
  100 pages fit in memory - read one page of R after the other and distribute over 99 partitions (whenever a page of a partition is full, move it to the disk).
  In the average, each partition them contains about 10.1 ($\rightarrow$ 11) pages; last page of each partition is not completely filled.
  maximum 1000 (read) + 1089 (write) = 2099.
- Same for S. Get 99 partitions of average 5.1 ($\rightarrow 6$) pages
  (maximum 500 + 594 = 1094 accesses).
- now there are 99 corresponding pairs of partitions (one from R, one from S).
  Join each pair: read $11 + 6$ pages per partition and process them.
  (about $99 \cdot (11 + 6) = 1683$ accesses).

Overall: 4876 accesses.

Solution to Exercise 4.6

First, compute all pairs of european countries. Note that the answer sould be their names, thus also the *Country* relation is needed.



- $cond =$ "enc.country=country.code" ,
- $eur =$ "enc1.continent='europe' and enc2.continent='europe'" ,
- $seas$ expresses that both countries border the same set of seas.
  It is a correlated subquery;
- add suitable projections;
- move $\sigma[eur]$ downwards both sides directly to *encompasses*;
- obviously, both subtrees of $\times$ are identical.

Solution to Exercise 4.6 (cont'd)

- $\sigma[seas(C_1, C_2)]$ is a correlated subquery that takes two country codes as input:

$$\begin{aligned}\sigma[seas(C_1, C_2)] &= seas(C_1) - seas(C_2) = \emptyset \wedge seas(C_2) - seas(C_1) = \emptyset \\ &= (seas(C_1) - seas(C_2)) \cup (seas(C_2) - seas(C_1)) = \emptyset\end{aligned}$$

- $seas(C) = \pi[sea](\sigma[country = C](geo\_sea))$
- for each country, $seas(C)$ is computed only once and then reused.

Resulting SQL skeleton (using subqueries in the FROM clause):

```
SELECT ...
FROM (SELECT european countries) as C1,
     (SELECT european countries) as C2
WHERE σ[seas(C1, C2)]
```


Solution to Exercise 4.7

- $\pi[abbrev](\sigma[all\_continents(org)])(organization)$ where $\sigma[all\_continents(org)]$ is true for

$$\{org \mid \forall cont : cont \text{ is a continent} \rightarrow org \text{ has a member on } cont\}$$

- convert $\forall$ into $\neg exists$:

$$\{org \mid \neg\exists cont : cont \text{ is a continent and } org \text{ has no member on } cont\}$$

- Thus, $\sigma[all\_continents(org)]$ checks if

$$\pi[name](continent) - \pi[enc.continent]((\sigma[organization = org]isMember) \bowtie encompasses)$$

is empty.

Resulting SQL skeleton (uses a correlated subquery):

```
SELECT ...
FROM organization
WHERE NOT EXISTS
      ((SELECT continents)
       MINUS
       (SELECT continents where org has a member))
```