

Chapter 3

Relational Database Languages: Relational Algebra

We first consider only *query* languages.

Relational Algebra: Queries are expressions over operators and relation names.

Relational Calculus: Queries are special formulas of first-order logic with free variables.

SQL: Combination from algebra and calculus and additional constructs. Widely used DML for relational databases.

QBE: Graphical query language.

Deductive Databases: Queries are logical rules.

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RELATIONAL DATABASE LANGUAGES: COMPARISON AND OUTLOOK

Remark:

- Relational Algebra and (safe) Relational Calculus have the same expressive power. For every expression of the algebra there is an equivalent expression in the calculus, and vice versa.
- A query language is called **relationally complete**, if it is (at least) as expressive as the relational algebra.
- These languages are compromises between efficiency and expressive power; they are not computationally complete (i.e., they cannot simulate a Turing Machine).
- They can be embedded into host languages (e.g. C++ or Java) or extended (PL/SQL), resulting in full computational completeness.
- Deductive Databases (Datalog) are more expressive than relational algebra and calculus.

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3.1 Relational Algebra: Computations over Relations

Operations on Tuples – Overview Slide

Let $\mu \in \text{Dup}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

(Formal definition of μ see Slide 61)

- For $\emptyset \subset \bar{Y} \subseteq \bar{X}$, the expression $\mu[\bar{Y}]$ denotes the **projection** of μ to \bar{Y} .

Result: $\mu[\bar{Y}] \in \text{Dup}(\bar{Y})$ where $\mu[\bar{Y}](A) = \mu(A), A \in \bar{Y}$.

- A **selection condition** α (wrt. \bar{X}) is an expression of the form $A \theta B$ or $A \theta c$, or $c \theta A$ where $A, B \in \bar{X}$, $\text{dom}(A) = \text{dom}(B)$, $c \in \text{dom}(A)$, and θ is a **comparison operator** on that domain like e.g. $\{=, \neq, \leq, <, \geq, >\}$.

A tuple $\mu \in \text{Dup}(\bar{X})$ **satisfies** a selection condition α , if – according to α – $\mu(A) \theta \mu(B)$ or $\mu(A) \theta c$, or $c \theta \mu(A)$ holds.

These (atomic) selection conditions can be combined to formulas by using \wedge, \vee, \neg , and $(,)$.

- For $\bar{Y} = \{B_1, \dots, B_k\}$, the expression $\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]$ denotes the **renaming** of μ .

Result: $\mu[\dots, A_i \rightarrow B_i, \dots] \in \text{Dup}(\bar{Y})$ where $\mu[\dots, A_i \rightarrow B_i, \dots](B_i) = \mu(A_i)$ for $1 \leq i \leq k$.

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Let $\mu \in \text{Dup}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

Projection

For $\emptyset \subset \bar{Y} \subseteq \bar{X}$, the expression $\mu[\bar{Y}]$ denotes the **projection** of μ to \bar{Y} .

Result: $\mu[\bar{Y}] \in \text{Dup}(\bar{Y})$ where $\mu[\bar{Y}](A) = \mu(A), A \in \bar{Y}$.

projection to a given set of attributes

Example 3.1

Consider the relation schema $R(\bar{X}) = \text{continent}(\text{Name}, \text{Area}): \bar{X} = [\text{Name}, \text{Area}]$

and the tuple $\mu = \boxed{\text{Name} \rightarrow \text{“Asia”}, \text{Area} \rightarrow 4.50953\text{e}+07}$.

formally: $\mu(\text{Name}) = \text{“Asia”}, \mu(\text{Area}) = 4.5\text{E}7$

projection attributes: Let $\bar{Y} = [\text{Name}]$

Result: $\mu[\text{Name}] = \boxed{\text{Name} \rightarrow \text{“Asia”}}$

□

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Again, $\mu \in \text{Tuple}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

Selection

A **selection condition** α (wrt. \bar{X}) is an expression of the form $A \theta B$ or $A \theta c$, or $c \theta A$ where $A, B \in \bar{X}$, $\text{dom}(A) = \text{dom}(B)$, $c \in \text{dom}(A)$, and θ is a **comparison operator** on that domain like e.g. $\{=, \neq, \leq, <, \geq, >\}$.

A tuple $\mu \in \text{Tuple}(\bar{X})$ **satisfies** a selection condition α , if – according to α – $\mu(A) \theta \mu(B)$ or $\mu(A) \theta c$, or $c \theta \mu(A)$ holds.

yes/no-selection of tuples (without changing the tuple)

Example 3.2

Consider again the relation schema $R(\bar{X}) = \text{continent}(\text{Name}, \text{Area})$: $\bar{X} = [\text{Name}, \text{Area}]$.

Selection condition: $\text{Area} > 10.000.000$.

Consider again the tuple $\mu = \boxed{\text{Name} \rightarrow \text{“Asia”}, \text{Area} \rightarrow 4.50953\text{e}+07}$.

formally: $\mu(\text{Name}) = \text{“Asia”}$, $\mu(\text{Area}) = 4.5E7$

check: $\mu(\text{Area}) > 10.000.000$

Result: yes. □

These (atomic) selection conditions can be combined to formulas by using \wedge , \vee , \neg , and $(,)$.

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Let $\mu \in \text{Tuple}(\bar{X})$ where $\bar{X} = \{A_1, \dots, A_k\}$.

Renaming

For $\bar{Y} = \{B_1, \dots, B_k\}$, the expression $\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]$ denotes the **renaming** of μ .

Result: $\mu[\dots, A_i \rightarrow B_i, \dots] \in \text{Tuple}(\bar{Y})$ where $\mu[\dots, A_i \rightarrow B_i, \dots](B_i) = \mu(A_i)$ for $1 \leq i \leq k$.

renaming of attributes (without changing the tuple)

Example 3.3

Consider (for a tuple of the table $R(\bar{X}) = \text{encompasses}(\text{Country}, \text{Continent}, \text{Percent})$):

$\bar{X} = [\text{Country}, \text{Continent}, \text{Percent}]$.

Consider the tuple $\mu = \boxed{\text{Country} \rightarrow \text{“R”}, \text{Continent} \rightarrow \text{“Asia”}, \text{Percent} \rightarrow 80}$.

formally: $\mu(\text{Country}) = \text{“R”}$, $\mu(\text{Continent}) = \text{“Asia”}$, $\mu(\text{Percent}) = 80$

Renaming: $\bar{Y} = [\text{Code}, \text{Name}, \text{Percent}]$

Result: a new tuple $\mu[\text{Country} \rightarrow \text{Code}, \text{Continent} \rightarrow \text{Name}, \text{Percent} \rightarrow \text{Percent}] =$

$\boxed{\text{Code} \rightarrow \text{“R”}, \text{Name} \rightarrow \text{“Asia”}, \text{Percent} \rightarrow 80}$ that now fits into the schema $\text{new_encompasses}(\text{Code}, \text{Name}, \text{Percent})$. □

The usefulness of renaming will become clear later ...

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EXPRESSIONS IN THE RELATIONAL ALGEBRA

What is an algebra?

- An algebra consists of a "domain" (i.e., a set of "things"), and a set of operators.
- Operators map elements of the domain to other elements of the domain.
- Each of the operators has a "semantics", that is, a definition how the result of applying it to some input should look like.
- **Algebra expressions** are built over basic constants and operators (inductive definition).

Relational Algebra

- The "domain" consists of all relations (over arbitrary sets of attributes).
- The operators are then used for combining relations, and for describing computations - e.g., in SQL.
- **Relational algebra expressions** are defined inductively over relations and operators.
- Relational algebra expressions define queries against a relational database.

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INDUCTIVE DEFINITION OF EXPRESSIONS

Atomic Expressions

- For an arbitrary attribute A and a constant $a \in \text{dom}(A)$, the **constant relation** $A : \{a\}$ is an algebra expression.

Format: $[A]$

Result relation: $\{a\}$

A:{a}
A
a

- Given a database schema $\mathbf{R} = \{R_1(\bar{X}_1), \dots, R_n(\bar{X}_n)\}$, every relation name R_i is an algebra expression.

Format of R_i : \bar{X}_i

Result relation (wrt. a given database state \mathcal{S}): the relation $\mathcal{S}(R_i)$ that is currently stored in the database.

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Structural Induction: Applying an Operator

- takes one or more input relations in_1, in_2, \dots
- produces a result relation out :
 - out has a **format**, depends on the formats of the input relations.
 - out is a relation, i.e., it contains some tuples, depends on the content of the input relations.

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BASE OPERATORS

Let \bar{X}, \bar{Y} formats and $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$ relations over \bar{X} and \bar{Y} .

Union

Assume $r, s \in \text{Rel}(\bar{X})$.

Result format of $r \cup s$: \bar{X}

Result relation: $r \cup s = \{\mu \in \text{Tup}(\bar{X}) \mid \mu \in r \text{ or } \mu \in s\}$.

$$r = \begin{array}{ccc} \hline A & B & C \\ a & b & c \\ d & a & f \\ c & b & d \end{array}$$
$$s = \begin{array}{ccc} \hline A & B & C \\ b & g & a \\ d & a & f \end{array}$$
$$r \cup s = \begin{array}{ccc} \hline A & B & C \\ a & b & c \\ d & a & f \\ c & b & d \\ b & g & a \end{array}$$

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Set Difference

Assume $r, s \in \text{Rel}(\bar{X})$.

Result format of $r \setminus s$: \bar{X}

Result relation: $r \setminus s = \{\mu \in r \mid \mu \notin s\}$.

$$r = \begin{array}{c|ccc} & A & B & C \\ \hline a & & b & c \\ d & & a & f \\ c & & b & d \end{array}$$

$$s = \begin{array}{c|ccc} & A & B & C \\ \hline b & & g & a \\ d & & a & f \end{array}$$

$$r \setminus s = \begin{array}{c|ccc} & A & B & C \\ \hline a & & b & c \\ c & & b & d \end{array}$$

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Projection

Assume $r \in \text{Rel}(\bar{X})$ and $\bar{Y} \subseteq \bar{X}$.

Result format of $\pi[\bar{Y}](r)$: \bar{Y}

Result relation: $\pi[\bar{Y}](r) = \{\mu[\bar{Y}] \mid \mu \in r\}$.

Example 3.4

Continent	
Name	Area
Europe	9562489.6
Africa	3.02547e+07
Asia	4.50953e+07
America	3.9872e+07
Australia	8503474.56

Let $\bar{Y} = [Name]$

$\mu_1[Name] = \boxed{Name \rightarrow "Europe"}$

$\mu_2[Name] = \boxed{Name \rightarrow "Africa"}$

$\mu_3[Name] = \boxed{Name \rightarrow "Asia"}$

$\mu_4[Name] = \boxed{Name \rightarrow "America"}$

$\mu_5[Name] = \boxed{Name \rightarrow "Australia"}$

$\pi[Name](\text{Continent})$
Name
Europe
Africa
Asia
America
Australia

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Selection

Assume $r \in \text{Rel}(\bar{X})$ and a selection condition α over \bar{X} .

Result format of $\sigma[\alpha](r)$: \bar{X}

Result relation: $\sigma[\alpha](r) = \{\mu \in r \mid \mu \text{ satisfies } \alpha\}$.

Example 3.5

Continent	
Name	Area
Europe	9562489.6
Africa	3.02547e+07
Asia	4.50953e+07
America	3.9872e+07
Australia	8503474.56

Let $\alpha = \text{"Area} > 10.000.000\text{"}$

$\mu_1(\text{Area}) > 10.000.000?$ – no

$\mu_2(\text{Area}) > 10.000.000?$ – yes

$\mu_3(\text{Area}) > 10.000.000?$ – yes

$\mu_4(\text{Area}) > 10.000.000?$ – yes

$\mu_5(\text{Area}) > 10.000.000?$ – no

$\sigma[\text{Area} > 10E6](\text{Continent})$	
Name	Area
Africa	3.02547e+07
Asia	4.50953e+07
America	3.9872e+07

□

Renaming

Assume $r \in \text{Rel}(\bar{X})$ with $X = [A_1, \dots, A_k]$ and a renaming $[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]$.

Result format of $\rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k](r)$: $[B_1, \dots, B_k]$

Result relation: $\rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k](r) = \{\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k] \mid \mu \in r\}$.

Example 3.6

Consider the renaming of the table *encompasses*(Country, Continent, Percent):

$\bar{X} = [\text{Country}, \text{Continent}, \text{Percent}]$

Renaming: $\bar{Y} = [\text{Code}, \text{Name}, \text{Percent}]$

$\rho[\text{Country} \rightarrow \text{Code}, \text{Continent} \rightarrow \text{Name}, \text{Percent} \rightarrow \text{Percent}](\text{encompasses})$		
Code	Name	Percent
R	Europe	20
R	Asia	80
D	Europe	100
⋮	⋮	⋮

□

(Natural) Join

Assume $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$ for arbitrary \bar{X}, \bar{Y} .

Convention: Instead of $\bar{X} \cup \bar{Y}$, we also write \overline{XY} .

for two tuples $\mu_1 = \boxed{v_1, \dots, v_n}$ and $\mu_2 = \boxed{w_1, \dots, w_m}$, $\mu_1\mu_2 := \boxed{v_1, \dots, v_n, w_1, \dots, w_m}$.

Result format of $r \bowtie s$: \overline{XY} .

Result relation: $r \bowtie s = \{\mu \in \text{Dup}(\overline{XY}) \mid \mu[\bar{X}] \in r \text{ and } \mu[\bar{Y}] \in s\}$.

Motivation

Simplest Case: $\bar{X} \cap \bar{Y} = \emptyset \Rightarrow$ Cartesian Product $r \bowtie s = r \times s$

$r \times s = \{\mu_1\mu_2 \in \text{Dup}(\overline{XY}) \mid \mu_1 \in r \text{ and } \mu_2 \in s\}$.

$r =$	$\begin{array}{c c} A & B \\ \hline 1 & 2 \\ 4 & 5 \end{array}$	$s =$	$\begin{array}{c c} C & D \\ \hline a & b \\ c & d \\ e & f \end{array}$	$r \bowtie s =$	$\begin{array}{c c c c} A & B & C & D \\ \hline 1 & 2 & a & b \\ 1 & 2 & c & d \\ 1 & 2 & e & f \\ 4 & 5 & a & b \\ 4 & 5 & c & d \\ 4 & 5 & e & f \end{array}$
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Example 3.7 (Cartesian Product of Continent and Encompasses)

Continent × encompasses				
Name	Area	Continent	Country	Percent
Europe	9562489.6	Europe	Germany	100
Europe	9562489.6	Europe	Russia	20
Europe	9562489.6	Asia	Russia	80
Europe	9562489.6	:	:	:
Africa	3.02547e+07	Europe	Germany	100
Africa	3.02547e+07	Europe	Russia	20
Africa	3.02547e+07	Asia	Russia	80
Africa	3.02547e+07	:	:	:
Asia	4.50953e+07	Europe	Germany	100
Asia	4.50953e+07	Europe	Russia	20
Asia	4.50953e+07	Asia	Russia	80
Asia	4.50953e+07	:	:	:
:	:	:	:	:

Back to the Natural Join

General case $\bar{X} \cap \bar{Y} \neq \emptyset$: shared attribute names constrain the result relation.

Again the definition: $r \bowtie s = \{\mu \in \text{Dup}(\overline{XY}) \mid \mu[\bar{X}] \in r \text{ and } \mu[\bar{Y}] \in s\}$.

(Note: this implies that the tuples $\mu_1 := \mu[\bar{X}] \in r$ and $\mu_2 := \mu[\bar{Y}] \in s$ coincide in the shared attributes $\bar{X} \cap \bar{Y}$)

Example 3.8

Consider *encompasses*(country,continent,percent) and *isMember*(organization,country,type):

<i>encompasses</i>			<i>isMember</i>		
Country	Continent	Percent	Organization	Country	Type
R	Europe	20	EU	D	member
R	Asia	80	UN	D	member
D	Europe	100	UN	R	member
:	:	:	:	:	:

$$\begin{aligned} \text{encompasses} \bowtie \text{isMember} = \{ \mu \in \text{Dup}(\text{country, cont, perc, org, type}) \mid \\ \mu[\text{country, cont, perc}] \in \text{encompasses} \text{ and } \mu[\text{org, country, type}] \in \text{isMember} \} \end{aligned}$$

□

Example 3.8 (Continued)

$$\begin{aligned} \text{encompasses} \bowtie \text{isMember} = \{ \mu \in \text{Dup}(\text{country, cont, perc, org, type}) \mid \\ \mu[\text{country, cont, perc}] \in \text{encompasses} \text{ and } \mu[\text{org, country, type}] \in \text{isMember} \} \end{aligned}$$

start with $(R, \text{Europe}, 20) \in \text{encompasses}$.

check which tuples in *isMember* match:

$(UN, R, \text{member}) \in \text{isMember}$ matches:

result: $(R, \text{Europe}, 20, UN, \text{member})$ belongs to the result.

(some more matches ...)

continue with $(R, \text{Asia}, 80) \in \text{encompasses}$.

$(UN, R, \text{member}) \in \text{isMember}$ matches:

result: $(R, \text{Asia}, 80, UN, \text{member})$ belongs to the result.

(some more matches ...)

continue with $(D, \text{Europe}, 100) \in \text{encompasses}$.

$(EU, D, \text{member}) \in \text{isMember}$ matches:

result: $(D, \text{Europe}, 100, EU, \text{member})$ belongs to the result.

$(UN, D, \text{member}) \in \text{isMember}$ matches:

result: $(D, \text{Europe}, 100, UN, \text{member})$ belongs to the result.

(some more matches ...)

□

Example 3.8 (Continued)

Result:

<i>encompasses</i> × <i>isMember</i>				
<i>Country</i>	<i>Continent</i>	<i>Percent</i>	<i>Organization</i>	<i>Type</i>
<i>R</i>	<i>Europe</i>	<i>20</i>	<i>UN</i>	<i>member</i>
<i>R</i>	<i>Europe</i>	<i>20</i>	<i>:</i>	<i>:</i>
<i>R</i>	<i>Asia</i>	<i>80</i>	<i>UN</i>	<i>member</i>
<i>R</i>	<i>Asia</i>	<i>80</i>	<i>:</i>	<i>:</i>
<i>D</i>	<i>Europe</i>	<i>100</i>	<i>UN</i>	<i>member</i>
<i>D</i>	<i>Europe</i>	<i>100</i>	<i>EU</i>	<i>member</i>
<i>D</i>	<i>Europe</i>	<i>100</i>	<i>:</i>	<i>:</i>
<i>:</i>	<i>:</i>	<i>:</i>	<i>:</i>	<i>:</i>

□

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Example 3.9 (and Exercise)

Consider the expression

$continent \bowtie \rho[Country \rightarrow Code, Continent \rightarrow Name, Percent \rightarrow Percent](encompasses)$

□

Functionalities of the Join

- Combining relations
- Selective functionality: only matching tuples survive
(consider joining cities and organizations on headquarters)

DERIVED OPERATORS

Intersection

Assume $r, s \in \text{Rel}(\bar{X})$.

Then, $r \cap s = \{\mu \in \text{Tup}(\bar{X}) \mid \mu \in r \text{ and } \mu \in s\}$.

Theorem 3.1

Intersection can be expressed by Difference: $r \cap s = r \setminus (r \setminus s)$.

□

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θ -Join

Combination of **Cartesian Product** and **Selection**:

Assume $r \in \text{Rel}(\bar{X})$, and $s \in \text{Rel}(\bar{Y})$, such that $\bar{X} \cap \bar{Y} = \emptyset$, and $A \theta B$ a selection condition.

$$r \bowtie_{A\theta B} s = \{\mu \in \text{Tup}(\overline{XY}) \mid \mu[\bar{X}] \in r, \mu[\bar{Y}] \in s \text{ and } \mu \text{ satisfies } A\theta B\} = \sigma[A\theta B](r \times s).$$

Equi-Join

θ -join that uses the “=”-predicate.

Example 3.10 (and Exercise)

Consider again Example 3.7:

Continent \times *encompasses* contained tuples that did not really make sense.

$(\text{Continent} \times \text{encompasses})_{\text{continent}=\text{name}}$ would be more useful.

Furthermore, consider

$\pi[\text{continent}, \text{area}, \text{code}, \text{percent}]((\text{Continent} \times \text{encompasses})_{\text{continent}=\text{name}})$:

- removes the - now redundant - “name” column,
- is equivalent to the natural join $(\rho[\text{name} \rightarrow \text{continent}]\text{continent}) \bowtie \text{encompasses}$. □

Semi-Join

- recall: joins combine, but are also selective
- semi-join acts like a selection on a relation r :
selection condition not given as a boolean formula on the attributes of r , but by “looking into” another relation (a subquery)

Assume $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$ such that $\bar{X} \cap \bar{Y} \neq \emptyset$.

Result format of $r \ltimes s$: \bar{X}

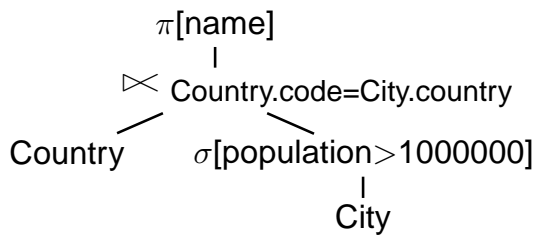
Result relation: $r \ltimes s = \pi[\bar{X}](r \bowtie s)$

The semi-join $r \ltimes s$ does *not* return the join, but checks which tuples of r “survive” the join with s (i.e., “which find a counterpart in s wrt. the shared attributes”):

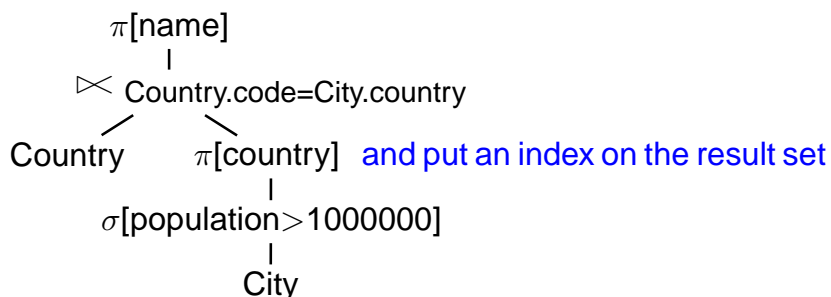
- Used with subqueries: (main query) \ltimes (subquery)
- $r \ltimes s \subseteq r$
- Used for optimizing the evaluation of joins (often in combination with indexes).

Semi-Join: Example

Give the names of all countries where a city with at least 1.000.000 inhabitants is located:



- Have a short look “inside” the subquery, but don’t actually use it:
- look only if there is a big city in this country.
- “if the country code is in the set of country codes ...”:



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Outer Join

- Join is the operator for combining relations

Example 3.11

- *Persons work in divisions of a company, tools are assigned to the divisions:*

Works	
Person	Division
John	Production
Bill	Production
John	Research
Mary	Research
Sue	Sales

Tools	
Division	Tool
Production	hammer
Research	pen
Research	computer
Admin.	typewriter

Works \bowtie Tools		
Person	Division	Tool
John	Production	hammer
Bill	Production	hammer
John	Research	pen
John	Research	computer
Mary	Research	pen
Mary	Research	computer

- *join contains no tuple that describes Sue*
- *join contains no tuple that describes the administration or sales division*
- *join contains no tuple that shows that there is a typewriter*

□

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Outer Join

Assume $r \in \text{Rel}(\bar{X})$ and $s \in \text{Rel}(\bar{Y})$.

Result format of $r \bowtie s$: \overline{XY}

The outer join extends the “inner” join with all tuples that have no counterpart in the other relation (filled with null values):

Example 3.12 (Outer Join)

Consider again Example 3.11

Works \bowtie Tools		
Person	Division	Tool
John	Production	hammer
Bill	Production	hammer
John	Research	pen
John	Research	computer
Mary	Research	pen
Mary	Research	computer
Sue	Sales	NULL
NULL	Admin	typewriter

Works \times Tools	
Person	Division
John	Production
Bill	Production
John	Research
Mary	Research

Works \times Tools	
Division	Tool
Production	hammer
Research	pen
Research	computer

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Formally, the result relation is defined as follows:

$J = r \times s$ — take the (“inner”) join as base

$r_0 = r \setminus \pi[\bar{X}](J) = r \setminus (r \times s)$ — r -tuples that “are missing”

$s_0 = s \setminus \pi[\bar{Y}](J) = s \setminus (r \times s)$ — s -tuples that “are missing”

$Y_0 = \bar{Y} \setminus \bar{X}$, $X_0 = \bar{X} \setminus \bar{Y}$

Let $\mu_1 \in \text{Tuple}(Y_0)$, $\mu_2 \in \text{Tuple}(X_0)$ such that μ_1, μ_2 consist only of *null* values

$$r \bowtie s = J \cup (r_0 \times \{\mu_1\}) \cup (s_0 \times \{\mu_2\}).$$

Example 3.12 (Continued)

For the above example,

$J = \text{Works} \times \text{Tools}$

$r_0 = [\text{“Sue”, “Sales”}]$, $s_0 = [\text{“Admin”, “Typewriter”}]$

$Y_0 = \text{Tool}$, $X_0 = \text{Person}$

$$\mu_1 = \begin{array}{|c|} \hline \text{Tool} \\ \hline \text{null} \\ \hline \end{array} \quad \mu_2 = \begin{array}{|c|} \hline \text{Person} \\ \hline \text{null} \\ \hline \end{array}$$

$$r_0 \times \{\mu_1\} = \begin{array}{|c|c|c|} \hline \text{Person} & \text{Division} & \text{Tool} \\ \hline \text{Sue} & \text{Sales} & \text{null} \\ \hline \end{array}$$

$$s_0 \times \{\mu_2\} = \begin{array}{|c|c|c|} \hline \text{Person} & \text{Division} & \text{Tool} \\ \hline \text{null} & \text{Admin} & \text{Typewriter} \\ \hline \end{array} \quad \square$$

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Generalized Natural Join

Assume $r_i \subseteq \text{ Tup}(\bar{X}_i)$.

Result format: $\cup_{i=1}^n \bar{X}_i$

Result relation: $\bowtie_{i=1}^n r_i = \{\mu \in \text{ Tup}(\cup_{i=1}^n \bar{X}_i) \mid \mu[\bar{X}_i] \in r_i\}$

Exercise 3.1

Prove that the natural join is associative (which makes the generalized natural join well-defined):

$$\begin{aligned} \bowtie_{i=1}^n r_i &= (((\dots((r_1 \bowtie r_2) \bowtie r_3) \bowtie \dots) \bowtie r_n)) \\ &= (r_1 \bowtie (r_2 \dots (r_{n-1} \bowtie r_n) \dots)) \end{aligned}$$

□

Relational Division

Assume $r \in \text{ Rel}(\bar{X})$ and $s \in \text{ Rel}(\bar{Y})$ such that $\bar{Y} \subsetneq \bar{X}$.

Result format of $r \div s$: $\bar{Z} = \bar{X} \setminus \bar{Y}$.

The result relation $r \div s$ is specified as “all \bar{Z} -values that occur in $\pi[\bar{Z}](r)$, with the additional condition that they occur in r together with **each of the \bar{Y} values that occur in s** ”.

Formally,

$$r \div s = \{\mu \in \text{ Tup}(\bar{Z}) \mid \{\mu\} \times s \subseteq r\} = \pi[\bar{Z}](r) \setminus \pi[\bar{Z}]((\pi[\bar{Z}](r) \times s) \setminus r).$$

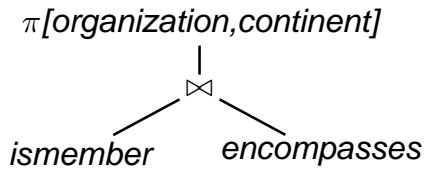
this implies that $\mu \in \pi[\bar{Z}](r)$

- Simple observation: $\pi[\bar{Z}](r) \supseteq r \div s$.
This constrains the set of possible results.
- Often, \bar{Z} and \bar{Y} correspond to the keys of relations that represent the instances of entity types.

Example 3.13 (Relational Division)

Compute those organizations that have at least one member on each continent:

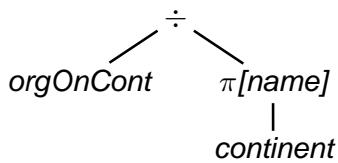
First step: which organizations have (some) member on which continents:



```
SELECT DISTINCT i.organization, e.continent
FROM ismember i, encompasses e
WHERE i.country=e.country
ORDER by 1
```

orgOnCont	
organization	continent
UN	Europe
UN	Asia
UN	America
UN	Africa
UN	Australia
NATO	Europe
NATO	America
NATO	Asia
:	:

Example 3.13 (Cont'd)



$$r(\bar{X}), s(\bar{Y}), \bar{Z} := \bar{X} \setminus \bar{Y}$$

$$r \div s = \{\mu \in \text{Dup}(\bar{Z}) \mid \{\mu\} \times s \subseteq r\}$$

$$\bar{X} = [\text{organization}, \text{continent}]$$

$$\bar{Y} = [\text{continent}]$$

Thus, $\bar{Z} = [\text{organization}]$.

orgOnCont	
organization	continent
UN	Europe
UN	Asia
UN	America
UN	Africa
UN	Australia
NATO	Europe
NATO	America
NATO	Asia
:	:

$\pi[\text{name}](\text{continent})$
continent
Asia
Europe
Australia
America
Africa

- UN: occurs with each continent in orgOnCont
⇒ belongs to the result.
- NATO: does not occur with each continent in orgOnCont
⇒ does not belong to the result.

Example 3.13 (Cont'd)

Consider again the formal algebraic characterization of Division:

$$r \div s = \{\mu \in \text{Tup}(\bar{Z}) \mid \{\mu\} \times s \subseteq r\} = \pi[\bar{Z}](r) \setminus \pi[\bar{Z}]((\pi[\bar{Z}](r) \times s) \setminus r).$$

1. $r = \text{orgOnCont}$, $s = \pi[\text{name}](\text{continent})$, $Z = \text{Country}$.
2. $(\pi[\bar{Z}](r) \times s)$ contains all tuples of organizations with each of the continents, e.g., $(\text{NATO}, \text{Europe})$, $(\text{NATO}, \text{Asia})$, $(\text{NATO}, \text{America})$, $(\text{NATO}, \text{Africa})$, $(\text{NATO}, \text{Australia})$.
3. $((\pi[\bar{Z}](r) \times s) \setminus r)$ contains all such tuples which are not "valid", e.g., $(\text{NATO}, \text{Africa})$.
4. projecting this to the organizations yields all those organizations where a non-valid tuple has been generated in (2), i.e., that have no member on some continent (e.g., NATO).
5. $\pi[\bar{Z}](r)$ is the list of all organizations ...
6. ... subtracting those computed in (4) yields those that have a member on each continent. \square

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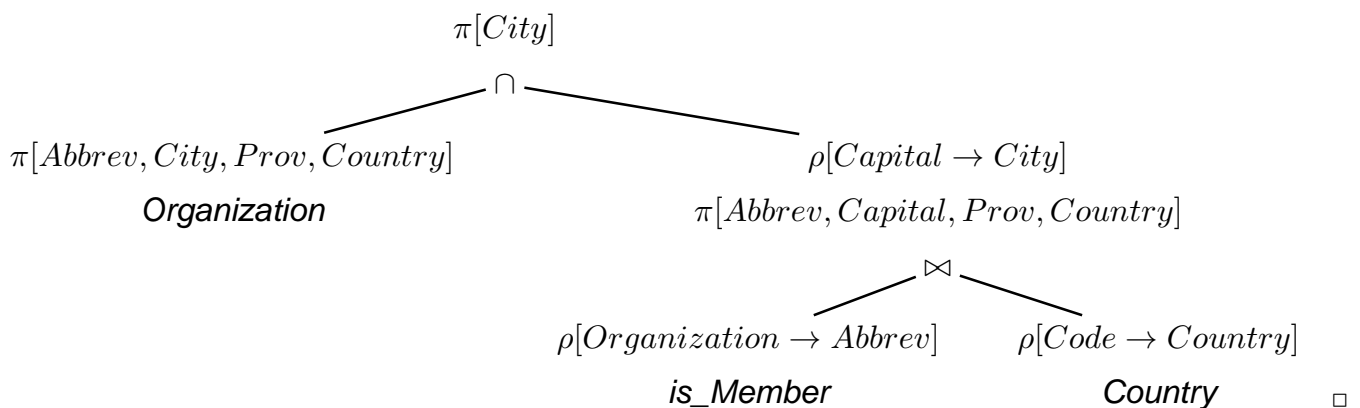
EXPRESSIONS

- inductively defined: combining expressions by operators

Example 3.14

The names of all cities where (i) headquarters of an organization are located, and (ii) that are capitals of a member country of this organization.

As a tree:



Note that there are many equivalent expressions. \square

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EXPRESSIONS IN THE RELATIONAL ALGEBRA AS QUERIES

Let $\mathbf{R} = \{R_1, \dots, R_k\}$ a set of relation schemata of the form $R_i(\bar{X}_i)$. As already described, an **database state** to \mathbf{R} is a **structure** \mathcal{S} that maps every relation name R_i in \mathbf{R} to a relation $\mathcal{S}(R_i) \subseteq \text{Tuple}(\bar{X}_i)$

Every algebra expression Q defines a **query** against the state \mathcal{S} of the database:

- For given \mathbf{R} , Q is assigned a **format** Σ_Q (the format of the answer).
- For every database state \mathcal{S} , $\mathcal{S}(Q) \subseteq \text{Tuple}(\Sigma_Q)$ is a relation over Σ_Q , called the **answer set** for Q wrt. \mathcal{S} .
- $\mathcal{S}(Q)$ can be computed according to the inductive definition, starting with the innermost (atomic) subexpressions.
- Thus, the relational algebra has a **functional semantics**.

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SUMMARY: INDUCTIVE DEFINITION OF EXPRESSIONS

Atomic Expressions

- For an arbitrary attribute A and a constant $a \in \text{dom}(A)$, the **constant relation** $A : \{a\}$ is an algebra expression.
 $\Sigma_{A:\{a\}} = [A]$ and $\mathcal{S}(A : \{a\}) = A : \{a\}$
- Every relation name R is an algebra expression.
 $\Sigma_R = \bar{X}$ and $\mathcal{S}(R) = \mathcal{S}(R)$.

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SUMMARY (CONT'D)

Compound Expressions

Assume algebra expressions Q_1, Q_2 that define $\Sigma_{Q_1}, \Sigma_{Q_2}, \mathcal{S}(Q_1)$, and $\mathcal{S}(Q_2)$.

Compound algebraic expressions are now formed by the following rules (corresponding to the algebra operators):

Union

If $\Sigma_{Q_1} = \Sigma_{Q_2}$, then $Q = (Q_1 \cup Q_2)$ is the **union** of Q_1 and Q_2 .

$\Sigma_Q = \Sigma_{Q_1}$ and $\mathcal{S}(Q) = \mathcal{S}(Q_1) \cup \mathcal{S}(Q_2)$.

Difference

If $\Sigma_{Q_1} = \Sigma_{Q_2}$, then $Q = (Q_1 \setminus Q_2)$ is the **difference** of Q_1 and Q_2 .

$\Sigma_Q = \Sigma_{Q_1}$ and $\mathcal{S}(Q) = \mathcal{S}(Q_1) \setminus \mathcal{S}(Q_2)$.

Projection

For $\emptyset \neq \bar{Y} \subseteq \Sigma_{Q_1}$, $Q = \pi[\bar{Y}](Q_1)$ is the **projection** of Q_1 to the attributes in \bar{Y} .

$\Sigma_Q = \bar{Y}$ and $\mathcal{S}(Q) = \pi[\bar{Y}](\mathcal{S}(Q_1))$.

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INDUCTIVE DEFINITION OF EXPRESSIONS (CONT'D)

Selection

For a selection condition α over Σ_{Q_1} , $Q = \sigma[\alpha]Q_1$ is the **selection** from Q_1 wrt. α .

$\Sigma_Q = \Sigma_{Q_1}$ and $\mathcal{S}(Q) = \sigma[\alpha](\mathcal{S}(Q_1))$.

Natural Join

$Q = (Q_1 \bowtie Q_2)$ is the **(natural) join** of Q_1 and Q_2 .

$\Sigma_Q = \Sigma_{Q_1} \cup \Sigma_{Q_2}$ and $\mathcal{S}(Q) = \mathcal{S}(Q_1) \bowtie \mathcal{S}(Q_2)$.

Renaming

For $\Sigma_{Q_1} = \{A_1, \dots, A_k\}$ and $\{B_1, \dots, B_k\}$ a set of attributes, $\rho[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k]Q_1$ is the **renaming** of Q_1

$\Sigma_Q = \{B_1, \dots, B_k\}$ and $\mathcal{S}(Q) = \{\mu[A_1 \rightarrow B_1, \dots, A_k \rightarrow B_k] \mid \mu \in \mathcal{S}(Q_1)\}$.

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Example

Example 3.15

Professor(PNr, Name, Office), *Course*(CNr, Credits, CName)

teach(PNr, CNr), *examine*(PNr, CNr)

- For each professor (name) determine the courses he gives (CName).

$$\pi [\text{Name}, \text{CName}] ((\text{Professor} \bowtie \text{teach}) \bowtie \text{Course})$$

- For each professor (name) determine the courses (CName) that he teaches, but that he does not examine.

$$\begin{aligned} & \pi [\text{Name}, \text{CName}] ((\\ & (\pi [\text{Name}, \text{CNr}] (\text{Professor} \bowtie \text{teach})) \\ & \setminus \\ & (\pi [\text{Name}, \text{CNr}] (\text{Professor} \bowtie \text{examine})) \\ &) \bowtie \text{Course}) \end{aligned}$$

Simpler expression:

$$\pi [\text{Name}, \text{CName}] ((\text{Professor} \bowtie (\text{teach} \setminus \text{examine})) \bowtie \text{Course}) \quad \square$$

EQUIVALENCE OF EXPRESSIONS

Algebra expressions Q, Q' are called **equivalent**, $Q \equiv Q'$, if and only if for all structures \mathcal{S} , $\mathcal{S}(Q) = \mathcal{S}(Q')$.

Equivalence of expressions is the basis for **algebraic optimization**.

Let $\text{attr}(\alpha)$ the set of attributes that occur in a selection condition α , and Q, Q_1, Q_2, \dots expressions with formats X, X_1, \dots

Projections

- $\bar{Z}, \bar{Y} \subseteq \bar{X} \Rightarrow \pi[\bar{Z}](\pi[\bar{Y}](Q)) \equiv \pi[\bar{Z} \cap \bar{Y}](Q)$.
- $\bar{Z} \subseteq \bar{Y} \subseteq \bar{X} \Rightarrow \pi[\bar{Z}](\pi[\bar{Y}](Q)) \equiv \pi[\bar{Z}](Q)$.

Selections

- $\sigma[\alpha_1](\sigma[\alpha_2](Q)) \equiv \sigma[\alpha_2](\sigma[\alpha_1](Q)) \equiv \sigma[\alpha_1 \wedge \alpha_2](Q)$.
- $\text{attr}(\alpha) \subseteq \bar{Y} \subseteq \bar{X} \Rightarrow \pi[\bar{Y}](\sigma[\alpha](Q)) \equiv \sigma[\alpha](\pi[\bar{Y}](Q))$.

Joins

- $Q_1 \bowtie Q_2 \equiv Q_2 \bowtie Q_1$.
- $(Q_1 \bowtie Q_2) \bowtie Q_3 \equiv Q_1 \bowtie (Q_2 \bowtie Q_3)$.

EQUIVALENCE OF EXPRESSIONS (CONT'D)

Joins and other Operations

- $\text{attr}(\alpha) \subseteq \bar{X}_1 \cap \bar{X}_2 \Rightarrow \sigma[\alpha](Q_1 \bowtie Q_2) \equiv \sigma[\alpha](Q_1) \bowtie \sigma[\alpha](Q_2)$.
- $\text{attr}(\alpha) \subseteq \bar{X}_1, \text{attr}(\alpha) \cap \bar{X}_2 = \emptyset \Rightarrow \sigma[\alpha](Q_1 \bowtie Q_2) \equiv \sigma[\alpha](Q_1) \bowtie Q_2$.
- Assume $V \subseteq \overline{X_1 X_2}$ and let $W = \bar{X}_1 \cap \overline{V X_2}, U = \bar{X}_2 \cap \overline{V X_1}$.
Then, $\pi[V](Q_1 \bowtie Q_2) = \pi[V](\pi[W](Q_1) \bowtie \pi[U](Q_2))$;
- $\bar{X}_2 = \bar{X}_3 \Rightarrow Q_1 \bowtie (Q_2 \text{ op } Q_3) = (Q_1 \bowtie Q_2) \text{ op } (Q_1 \bowtie Q_3)$ where $\text{op} \in \{\cup, -\}$.

Exercise 3.2

Prove some of the equalities (use the definitions given on the “Base Operators” slide). □

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EXPRESSIVE POWER OF THE ALGEBRA

Transitive Closure

The transitive closure of a binary relation R , denoted by R^* is defined as follows:

$$\begin{aligned} R^1 &= R \\ R^{n+1} &= \{(a, b) \mid \text{there is an } s \text{ s.t. } (a, x) \in R^n \text{ and } (x, b) \in R\} \\ R^* &= \bigcup_{1.. \infty} R^n \end{aligned}$$

Examples:

- $\text{child}(x, y)$: $\text{child}^* = \text{descendant}$
- flight connections
- flows_into of rivers in MONDIAL

Theorem 3.2

There is no expression of the relational algebra that computes the transitive closure of arbitrary binary relations r . □

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EXAMPLES

Time to play. Perhaps postpone examples after comparison with SQL (next subsections)

Aspects

- join as “extending” operation (cartesian product – “all pairs of X and Y such that ...”)
- equijoin as “restricting” operation
- natural join/equijoin in many cases along key/foreign key relationships
- relational division (in case of queries of the style “return all X that are in a given relation with all Y such that ...”)