# Chapter 4 Internal Organization and Implementation

This section heavily relies on other subdisciplines of Practical Computer Science:

- System Structures, down to the physical level
- Operating Systems Aspects: Caching
- Algorithms (mainly: for joins) and Data Structures (tree indexes, hashing)

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#### PHYSICAL DATA ORGANIZATION

- the **conceptual schema** defines which data is described and its semantics.
- the physical schema defines the physical database where the data is actually stored.
   ⇒ efficiency
- system: the data is actually stored in **files**: data that semantically belongs together (a relation, a part of a relation (**hashing**), some relations (**cluster**)).
- additionally, there are files that contain auxiliary information (indexes).
- data is accessed **pagewise** or **blockwise** (typically, 4KB 8KB).
- each page contains some **records** (tuples). Records consist of **fields** that are of an elementary type, e.g., bit, integer, real, string, or pointer.

## DB SERVER ARCHITECTURE: SECONDARY STORAGE AND CACHING

runtime server system: accessed by user queries/updates

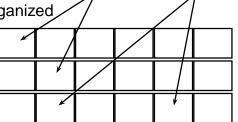
- parser: translates into algebra, determines the required relations + indexes
- file manager: determines the file/page where the requested data is stored
- buffer/cache manager: provides relevant data in the cache
- query/update processing: uses only the cache

Cache (main memory): pagewise organized

- Accessed pages are fetched into the cache
- pages are also changed in the cache
- and written to the database later ...

Secondary Storage (Harddisk): pagewise organized

- data pages with tuples
- index pages with tree indexes (see later)
- database log etc. (see later)



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## **DATABASE ACCESS MECHANISM**

Records must be loaded from (and written to) the secondary memory for processing:

- the **file manager** determines the page where the record is stored.
- the buffer/cache manager is responsible to provide the page in the buffer (buffer management):
  - maintains a **pool** of pages (organized as frames).
     for every page, it is stored if the page has been changed, how often/frequently it has been used, and if it is currently used by transactions
  - if the required page is not in the cache, some stored page is replaced (if it has been changed, it must be written to the secondary memory)
- complex prefetching strategies, based on knowledge about transactions.
   [see lecture on Operating Systems]
- for now, it is sufficient to note that we have to deal with pagewise access.

## Storage of Files, Pages, and Records

- Inside a file, every tuple/record has a tuple identifier of the form (p, n) where p is the page number and n is its index inside the page.
  - Each page then contains a **directory** that assigns a physical address to each n.
- memory management for deleted records
- different strategies for fixed-length and variable-length records
- ... so far to the physical facts ...

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# 4.1 Efficient Data Access

- efficiency depends on the detailed organization and additional algorithms and data structures
- support generic operations:
  - Scan: all pages that contain records are read.
  - Equality Search: all records that satisfy some equality predicate are read.
     SELECT \* FROM City WHERE Country = "D";
  - Range Search: all records that satisfy some comparison predicate are read.
     SELECT \* FROM City WHERE Population > 100.000;
  - Modify, Delete: analogously
  - Insert: analogously: search for an appropriate place where to put the record.
- linear search (scan) ??
- Need for efficient searching (equality and/or range)

#### INDEXING

Indexes (for a file) are auxiliary structures that support special (non-linear) access paths

- Based on search keys
- not necessarily the relational "keys", but any combination of attributes
- a relation may have several search keys
- an *index* is a set of data entries on some pages together with efficient access mechanisms for locating an entry according to its search key value.
- different types of indexes, depending on the operations to be supported:
  - equality search
  - range search (ordered values)
  - search on small domains
- in general, joins by key/foreign-key references are supported by indexes.

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#### TREE INDEXES

This topic brings data structures and databases (= applications of data structures) together.

- introductory lecture "Computer Science I": store numbers in trees.
- databases: tree index over the values of a column of a relation
  - search tree based on the values (numbers, strings)
  - the tuples themselves are not stored in the tree
  - entries (or leaf entries only) hold the values and point to the respective tuples in the database
- special trees with higher degrees:
  - each node (of the size of a storage page) has multiple entries and multiple children.

#### B- AND B\*-TREES

A **B-tree** (R. Bayer & E.McCreight, 1970) of order  $(m, \ell)$ ,  $m \ge 3$ ,  $\ell \ge 1$ , is a search tree [see lecture on Algorithms and Data Structures]:

- the root is either a leaf or it has at least 2 children
- every inner node has at least  $\lceil m/2 \rceil$  and at most m children
- all leaves are on the same level (balanced tree), and hold at most  $\ell$  entries
- inner nodes have the form  $(p_0, k_1, p_1, k_2, p_2, \dots, k_n, p_n)$  where  $\lceil m/2 \rceil 1 \le n \le m-1$  and
  - $k_i$  are search key values, ordered by  $k_i < k_j$  for i < j
  - $p_i$  points to the i+1th child
  - all search key values in the left (i.e.,  $p_{i-1}$ ) subtree are less than the value of  $k_i$  (and all values in the right subtree are greater or equal)

B-trees are used for "simply" organizing items of an ordered set (e.g. for sorting) as an extension of binary search trees.

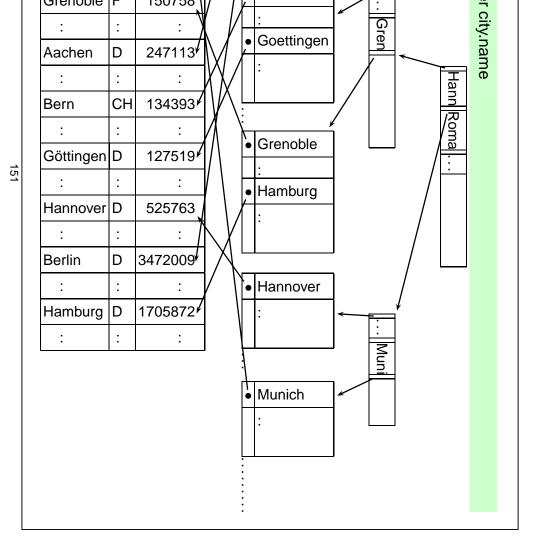
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## B\*-TREES

(sometimes also called B+-Trees)

A **B**\*-tree of order  $(m, \ell)$ ,  $m \ge 3$ ,  $\ell \ge 1$ , is closely related, except:

- they are intended to associate data with search key values:
- the inner nodes do not hold additional data, but are still intended to guide the search.
- The leaves are of the form  $([k_1, s_1], [k_2, s_2], \dots, [k_g, s_g])$  where  $g \leq \ell$ ,  $k_i$  is a search key value, and  $s_i$  is a data record or (in databases) a pointer to the corresponding record.



## **Properties**

- Let N the number of entries. Then, for the height h of the tree,  $h \leq \lceil log_{m/2}(2N/\ell) \rceil$  ( $\ell/2$  entries per leaf, inner nodes half filled) and  $h \geq \lceil log_m(N/\ell) \rceil$  ( $\ell$  entries/leaf, inner nodes completely filled)
- equality search needs h steps inside each of the inner nodes, binary search is applied
- if the leaves are connected by pointers, ordered sequential access (range search) is also supported
- insertions and modifications may be expensive (tree reorganization)

#### Use of B\*-Trees as Access Paths in Databases

- databases: cities stored in data files, index trees hold pointers to city records in their tree entries.
- separation between index files/pages and data files/pages.
- multiple search trees for each relation possible.

## Example

### **Example 4.1**

Consider the Mondial database with 3000 cities, with an index over the name. Assume the following sizes:

- every leaf (tuple) page contains 10 cities,
- every inner node contains 20 pointers

#### Then

- every inner node on the lowest level covers 200 cities
- every inner node on the second lowest level covers 4000 cities
- minimal: only one level of inner nodes
- maximal: two levels of inner nodes (nodes about only 2/3 filled)
- access every city with WHERE Name ="..." in 3 or 4 steps
- index on population, e.g., for WHERE Population > 1.000.000 ORDER BY Population
- realistic numbers: block size 4K: lowest level (keys+pointers to DB): 100 cities; inner nodes: 100 references.

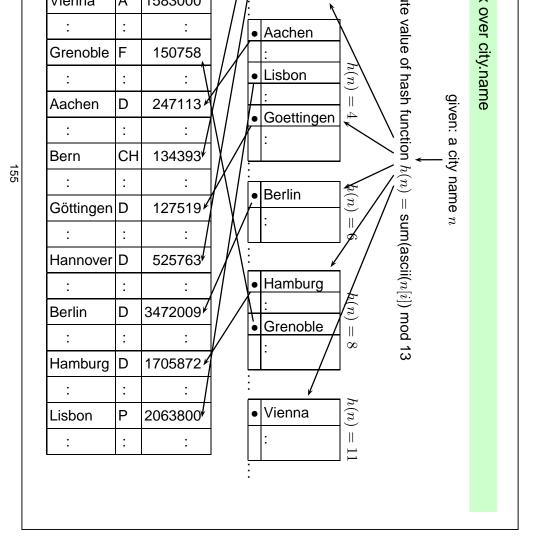
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# HASH-INDEX (DICTIONARY)

Hash index over the value of one or more columns ("hask key" – is not necessarily a key of the relation):

The values are distributed over k **tiles**.

- A hash function h is a function that maps each value to a tile number.
   operation: lookup(value)
- each tile holds pairs (key,pointer) to all tuples whose hash-key value is mapped to this tile;
- each tile consists of one or more pages.
- common technique: convert value to an integer i. i mod k gives the tile number.



#### Hash-Index

#### Example 4.2

Multi-attribute hash keys:

Consider a hash index on City.(name,province,country). h computes the sum of the ASCII numbers of the letters and takes the remainder modulo 111.

## **Properties:**

- equality search and insert in constant time (+ time for searching in the tile)
- does not support range queries or ordered output

#### **Comments:**

- maintenance of overflow pages: see Info III
- lookup inside each tile can be organized by a B-tree (kept on a single page)

## **BIT LISTS**

If the number of possible values of a search key value is small wrt. the number of tuples, **bit lists** are useful as indexes

Let a an attribute of the records stored in a file f, where k values of a exist. Then, a bit list index to f consists of k bit vectors  $B(v_i)$ ,  $i \le 1 \le k$ .

•  $B(v_i)(j) = 1$  if the jth record in f has the value  $v_i$  for the attribute a.

#### **Properties:**

access all tuples with a given value:
 without bit list: linear search over all pages
 with bit list: access bit list, and access those pages where a "1"-tuple is located.

• modifications: no problem

deletions: depends (if gaps are allowed on the pages)

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## Example 4.3

Consider the relation is\_member(organization, country, type) where type has only the values "member", "applicant", and "observer":

is_member		
Org.	Country	Туре
EU	D	member
UN	D	member
UN	СН	observer
UN	R	member
EU	PL	applicant
EU	CZ	applicant
:	:	:

Bit list for type=member: 1 1 0 1 0 0 ...

Bit list for type=observer: 0 0 1 0 0 0 . . .

Bit list for type=applicant: 0 0 0 0 1 1 ...

Bit list for org=EU: 1 0 0 0 1 1 . . .

Bit list for org=UN: 0 1 1 1 0 0 . . .

- Search for members of the UN: without bit list: linear search over all pages with bit list: access bit list, and access those pages where a 1-tuple is located.
- 2nd bit list on organization column: "members of UN" by logical "and" of bit lists.

## **CLUSTERING**

Data from the secondary storage is fetched by **pages**. Pages are cached in the main memory.

- keep data that semantically belongs together on the same pages
- obviously done for relations
- data of several relations  $R_1(\bar{X}_1), \dots, R_n(\bar{X}_n)$  can also be grouped to **clusters**:
  - choose a set  $\bar{Y} \subseteq \bar{X}_1 \cap \ldots \cap \bar{X}_n$  as cluster key.
  - combine relations by their  $\bar{Y}$ -values.
  - provides obvious advantages when evaluating joins over the cluster key.

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## Example 4.4

Consider the relation schemata

organization(name,abbrev,established,...) and

is\_member(organization, country, type).

The foreign key

 $is\_member.organization \rightarrow organization.abbrev$  is used as cluster key.

Organization		
Abbrev		
EU	Name	established
	European Union	07.02.1992
	Country	Туре
	D	member
	Α	member
	•	:
UN	Name	established
	United Nations	26.06.1945
	Country	Туре
	D	member
	СН	observer
	:	:

# 4.1.1 Algorithms and Data Structures: Basic Techniques

**Iteration:** all tuples in the input relations are processed iteratively. If possible, instead of the tuples themselves, an index can be used.

**Indexes:** selections and joins can be based on processing of an index for determining the tuples that satisfy the selection condition (a join condition is also a selection condition).

Indexes on single attributes: obvious.

## Indexes on multiple attributes:

- a hash index to a conjunction of equality predicates of the form field = value can be
  used if every field of the search key occurs exactly in one predicate together with a
  constant,
- a **tree index** to a conjunction of **comparison** predicates of the form **field**  $\theta$  **value** can be used if a prefix of the search key exists such that each field of the prefix occurs in exactly one predicate together with a constant.

Example: if (Country,Province,CityName) is the search key of an index, it can also be used as an index for the key's prefix (Country,Province).

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# 4.1.2 Implementation of Algebra Operations

#### SELECTION

Selection:  $\sigma[R.field \theta value]R$ .

- no index, unordered tuples: linear scan of the file
- no index, tuples ordered wrt. field: find the tuple(s) that satisfy " $field \ \theta \ value$ " (binary search) and process the these tuples.
- tree index: find the tuple(s) that satisfy " $field \ \theta \ value$ " using the tree index and process these tuples.
- hash: suitable only if  $\theta$  is equality.
- + strategies for compound selection conditions.

#### **PROJECTION**

Projection:  $\pi[field_1, field_2, ..., field_m]R$ .

Main problem: remove duplicates (relational algebra does not allow duplicates, SQL does).

- by sorting:
  - scan the file, create a new file with projected tuples.
  - sort the new file (over all fields,  $n \log n$ ).
  - scan the result, remove duplicates.
- by hash: scan the file, put the projected tuples into a hash table.
   duplicates all end up in the same tile (still in files).
   remove duplicates separately for each tile (iterating the hashing process with different functions until tile fits in main memory)
   collect the tiles in the output file.
  - ⇒ hashing not only as an index for search *structures*, but also as an *algorithm* for partitioning.

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#### JOIN

Consider an Equijoin:  $R \bowtie_{R,A=S,B} S$ .

Nested-loop-join

foreach tuple 
$$r \in R$$
 do 
$$\text{foreach tuple } s \in S \text{ do}$$
 
$$\text{if } r_A = s_B \text{ then add } [r,s] \text{ to result}$$

Very inefficient:

- assume: only one page of each relation fits into main memory
- m tuples of R per page, n tuples of S per page:  $|R|/m + |R| \cdot |S|/n$  filesystem accesses (for each tuple of R, loop over all pages of S).
- $|R| \cdot |S|$  comparisons.

obvious: if possible, process the smaller relation in inner loop to keep it in main memory (|R|/m + |S|/n filesystem accesses).

## Block-nested-loop-Join

Optimization by page-oriented strategy:

Divide S into blocks that fit in main memory and process each of the blocks separately.

- assume: only one page of each relation fits into main memory
- *m* tuples of *R* per page, *n* tuples of *S* per page:
  - take first page  $(R_1)$  of R and first page  $(S_1)$  of S;
  - combine *each* tuple of  $R_1$  with each tuple with  $S_1$ .
  - continue with  $R_1$  and second page  $(S_2)$  of S ... up to  $R_1$  with  $S_{last}$
  - continue with  $R_2$  and  $S_1$  etc.
- $|R|/m + |R|/m \cdot |S|/n$  filesystem accesses (for each page of R, loop over all pages of S).
- still  $|R| \cdot |S|$  comparisons.

## Further optimization:

when a page of S is loaded, generate a temporary index over the join attribute(s) that is used in the comparison (then, only matching pairs are considered).

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## Index-nested-loop-Join

If for one of the input relations there is an index over the join attribute, choose it as the inner relation.

- given index on S's join attribute
- loop over R, for each value access matching tuple(s) in S.
- |R|/m + |Result| filesystem accesses,
- up to  $|R|/m + |R| \cdot |S|$  filesystem accesses,
- maybe less efficient than block-nested-loop!

#### **Parallelization**

Divide one of the relations into partitions and process each of them on a separate processor.

## Sort-merge-join

Algorithm type: "Scan line"

- compute sorted indexes of both relations on the join attributes
   (if tree indexes on join attributes are available, simply use them)
- search for matches as follows:
  - proceed through the ordered indexes R and S stepwise, always doing a step in the index where the "smaller" value is.
- if a match is found:
  - access tuples and generate a result tuple.
  - check for tuples which have the same values of the join attributes (must immediately follow this match in both relations).
- |R|/m + |S|/n + |Result| filesystem accesses.
- much less, if indexes are already available or relations are stored in this order.

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#### Sort-merge-join: Algorithm

```
if R not sorted on attribute A, sort it;
if S not sorted on attribute B, sort it;
Tr := first tuple in R;
Ts := first tuple in S;
Gs := first tuple in S;
while Tr \neq eof and Ts \neq eof do {
       while Tr \neq eof and Tr.A < Ts.B do Tr = next tuple in R after Tr;
       while Ts \neq eof and Tr.A > Ts.B do Ts = next tuple in S after Ts;
       // now, Tr.A = Ts.B: match found
       while Tr \neq eof and Tr.A = Ts.B do {
              Gs := Ts;
              while Gs \neq eof and Gs.B = Tr.A do {
                    add [Tr, Gs] to result;
                    Gs = next tuple in S after Gs;
              Tr = next tuple in R after Tr;
       Ts := Gs:
}
```

## Hash-join

Algorithm type: "Divide and Conquer"

Partitioning (building) phase:

- Partition the smaller relation R by a hash function  $h_1$  applied to R.A.
- Partition the larger relation S by the same hash function applied to S.B. (into different hash tables)

Matching (probing) phase:

- potential(!) matches have been mapped to "face-to-face" partitions.
- thus, consider each pair of corresponding partitions:
- if partition of R does not fit into main memory, proceed recursively with another h.
- otherwise i.e., *R*-partition fits in main memory:
  - if corresponding S-partition also fits into main memory, compute the join.
  - otherwise proceed recursively with another (finer)  $h_2$  inside main memory and process S-partition pagewise.

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## **EXERCISE**

#### **Exercise 4.1**

Consider the join of two relations R and S wrt. the join condition  $R_i = S_j$ . R uses M pages with  $p_R$  tuples each, and S uses N pages with  $p_S$  tuples each. Let M = 1000,  $p_R = 100$ , N = 500,  $p_S = 80$ ; the cache can keep 100 pages. Compute the number of required I/O-operations for computing the join (without writing the result relation) for:

- (a) simple nested-loop-join,
- (b) pagewise nested-loop-join,
- (c) index-nested-loop-join (Index on  $S_i$ ),
- (d) sort-merge-join,
- (e) hash-join.

## **SEMIJOINS**

• Natural Semijoins or Condition Semijoins as subqueries:

$$R \bowtie [\mathsf{condition}(R_{i_1}, ...R_{i_1}, S_{i_1}...S_{i_\ell}]S$$

It is often sufficient to use an index over  $S_{i_1}...S_{i_\ell}$  instead of the whole relation.

• Semjoins as preparation for joins:  $R \bowtie S = (R \bowtie S) \bowtie S$  (and symmetrically

$$= R \bowtie (R \bowtie S)$$

Is that simpler? - Sometimes Yes!

- cf. Join  $R \bowtie S$  with if an existing index over the join attribute(s) in S: iterate over R and access S via index. No access, if not found.

This actually is  $(R \bowtie S) \bowtie S$ .

Consider  $R \bowtie S$  to be a hash join:

- First hash S, build an index over the join attribute(s), keep the index in main memory.
- iterate over R: for every tuple  $\mu$ , if  $\mu \in (R \bowtie S)$  (can be checked against the main memory index), then hash it.

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## **GROUPING AND AGGREGATION**

#### **General Structure:**

SELECT  $A_1, \ldots, A_n$  list of attributes

FROM  $R_1, \ldots, R_m$  list of relations WHERE F condition(s)

GROUP BY  $B_1, \ldots, B_k$  list of grouping attributes

HAVING G condition on groups

ORDER BY H sort order

- SUM, AVG, COUNT: linear scan necessary (need to consider all tuples)
- MIN, MAX: index can be used
- support for grouping (SQL: GROUP BY) using an index or a hash table

## Union, difference, intersection ⇔ cartesian product

- intersection and cartesian product are special cases of join
- union and difference: analogous to sort-merge-join or hash-join

## SITUATION-DEPENDENT OPTIMIZATION

Determining the optimal **execution plan** depends on **cost models**, **heuristics**, and the actual physical schema and the actual database contents:

- index structures
- statistics on data (i.e., to some extent state dependent)
  - cardinality of relations
  - distribution of values

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## **Notions**

The **selectivity** sel of operations can be used for estimating the size of the result relation:

Selection with condition p:

$$sel_p = \frac{|\sigma[p](R)|}{|R|}$$

Proportion of tuples that satisfy the selection condition.

- for p an equality test R.A = c where A is a key of R,  $sel_p = 1/|R|$ .
- if R.A distributes evenly on i different values,  $sel_p = 1/i$ .
- Join of R and S:

$$sel_{RS} = \frac{|R \bowtie S|}{|R \times S|} = \frac{|R \bowtie S|}{|R| \cdot |S|}$$

is the proportion of result tuples wrt. the cartesian product.

for  $R \bowtie_{R.A=S.B} S$  with A a key attribute,  $|R \bowtie_{R.A=S.B} S| \leq |S|$ , thus  $sel_{RS} \leq 1/|R|$ .

- optimal order of applying joins
- optimal order of applying selections

## APPLICATION-LEVEL ALGORITHMIC OPTIMIZATION

And never forget about using efficient algorithms for querying the database!

- analyze the problem from the algorithmic point of view
- before hacking

## **EXAMPLES**

Use the MONDIAL database for the following examples.

#### Example 4.5

Compute all pairs of european countries that are adjacent to the same set of seas.

#### Example 4.6

Compute all political organizations that have at least one member country on every continent (this operation is called relational division).

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# 4.1.3 Exercises and Examples

#### Solution to Exercise 4.1

R: 1000 pages, each of them with 100 tuples

S: 500 pages, each of them with 80 tuples

Cache: 100 pages

- a) simple nested loop join
  - outer loop: RLoad each page of R. For all tuples, iterate over S's pages.  $1000 \cdot (1 \text{ (load)} + 100 \text{ (tuples per page)} \cdot 500 \text{ (pages of } S)) = 50,001,000$
  - outer loop: SLoad each page of S. For all tuples, iterate over R's pages.  $500 \cdot (1 \text{ (load)} + 80 \text{ (tuples per page)} \cdot 1000 \text{ (pages of } R)) = 40,000,500$

## Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples

S: 500 pages, each of them with 80 tuples

Cache: 100 pages

- b) pagewise nested loop:
  - outer loop: R. Load each page of R. Combine all tuples of that page with all tuples from each page of S.

```
1000 \cdot (1 + 500 \text{ (pages of } S)) = 501,000
```

- outer loop:  $S. 500 \cdot (1 + 1000 \text{ (pages of } R)) = 505,000$
- b2) maximum-pages nested loop: load as many (first 99) pages of R as possible in the cache and join with one page of S after the other. Then continue with 2nd 99 pages.
  - 11 times ( $10 \cdot 99 \ R$  pages +  $1 \cdot 10 \ R$  pages) through all S pages. Overall:  $1000 + 11 \cdot 500 = 6500$ .
  - symmetric with (5  $\cdot$  99 + 1  $\cdot$  5) pages of S:

Overall:  $500 + 6 \cdot 1000 = 6500$ .

 can be algorithmically optimized by an on-the-fly index during the main-memory join: index the 99 pages, loop over the single page.

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## Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples

S: 500 pages, each of them with 80 tuples; Cache: 100 pages

- c) Index-nested-loop (index on  $S_i$ ): Assumptions:
  - every  $R_i$  matches average k = 4 S tuples times.
  - index over S already exists and can be kept in main memory (creating an index over S makes 500 accesses)

Iterate over R, for each tuple search for the  $S_j$  key and access the tuples

 $1000 \cdot (1 \text{ (load)} + 100 \cdot 4 \text{ (access tuples)}) = 401,000.$ 

Note: the number of page accesses depends on the number of results since for every actual result there is one page access.

In case that S is ordered wrt.  $S_j$ :  $1000 \cdot (1 \text{ (load)} + 100 \cdot 1 \text{ (access tuples)}) = 101,000.$ 

d) sort-merge: sort R according to R.i:

sort each 100 pages in place (Quicksort), store them:

10 (times)  $\cdot$  100 (pages)  $\cdot$  2 (read and write) = 2000

mergesort the sorted pages (linear) – read and write: 2000

sort S according to  $S_i$  analogously: 2000

merge-join of sorted relations: linear scan: 1000 + 500 = 1500

Overall: 7500

#### Solution to Exercise 4.1 (cont'd)

R: 1000 pages, each of them with 100 tuples

S: 500 pages, each of them with 80 tuples

Cache: 100 pages

## e) Hash-Join:

- Hash R:

100 pages fit in memory - read 34 pages and distribute over 60 partitions. Do this 30 times.

(about  $30 \cdot (34 \text{ (read)} + 60 \text{ (write)}) < 3000 \text{ accesses)}$ .

In the average, each partition them contains about 17 pages.

- Same for S. Get 60 partitions of average 9 pages. (about 1500 accesses).
- now we have 60 corresponding pairs of partitions (one from R, one from S). Join each pair: read 17 + 9 (overestimate) pages per partition and process them. (about  $60 \cdot (17 + 9) = 1560$  accesses).

Overall: 6060 accesses.

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#### Solution to Exercise 4.5

First, compute all pairs of european countries. Note that we are interested in their names, thus we need also the *Country* relation.

$$\sigma[seas] \\ \sigma[eur] \\ \\ \rho[enc \to enc1] \times \rho[enc \to enc2] \\ \bowtie_{cond} \\ \\ \text{encompasses} \quad \text{country} \quad \text{encompasses} \quad \text{country}$$

- cond = "enc.country=country.code".
- eur = "enc1.continent='europe' and enc2.continent='europe'".
- seas expresses that both countries border the same set of seas.
   It is a correlated subquery.
- add suitable projections.
- move  $\sigma[eur]$  downwards both sides directly to *encompasses*
- obviously, both subtrees of x are identical.

## Solution to Exercise 4.5 (cont'd)

•  $\sigma[seas(C_1, C_2)]$  is a correlated subquery that takes two country codes as input:

$$\sigma[seas(C_1, C_2)] = seas(C_1) - seas(C_2) = \emptyset \land seas(C_2) - seas(C_1) = \emptyset$$
$$= (seas(C_1) - seas(C_2)) \cup (seas(C_2) - seas(C_1)) = \emptyset$$

- $seas(C) = \pi[sea](\sigma[country = C](geo\_sea))$
- ullet for each country, seas(C) is computed only once and then reused.

Resulting SQL skeleton (using subqueries in the FROM clause):

```
SELECT ... FROM (SELECT european countries) as C1, (SELECT european countries) as C2 WHERE \sigma[seas(C_1,C_2)]
```

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#### Solution to Exercise 4.6

- $\pi[abbrev](\sigma[all\_continents(org)])(organization)$  where  $\sigma[all\_continents(org)]$  is true for  $\{org \mid \forall cont : cont \text{ is a continent} \rightarrow org \text{ has a member on } cont\}$
- convert  $\forall$  into  $\neg exists$ :

 $\{org \mid \neg \exists cont : cont \text{ is a continent and } org \text{ has no member on } cont\}$ 

• Thus,  $\sigma[all\_continents(org)]$  checks if  $\pi[name](continent) - \pi[enc.continent]((\sigma[organization = org]is\_member) \bowtie encompasses)$  is empty.

Resulting SQL skeleton (uses a correlated subquery):

```
SELECT ...

FROM organization

WHERE NOT EXISTS

((SELECT continents)

MINUS

(SELECT continents where org has a member))
```