2. Unit: SPARQL Formal Semantics

Exercise 2.1 (SPARQL Formal Semantics) Consider the SPARQL Formal Semantics.

a) Define a “null-tolerant join” for the relational algebra that acts like the $\bowtie$ of the SPARQL algebra.

b) Which SQL construct is similar to the “\” operator in the SPARQL algebra?

c) In the SPARQL algebra, OPT is expressed via left outer join, which is defined via “\” (while a corresponding MINUS does not exist in the SPARQL syntax).
Such a MINUS (cf. part (b) of this exercise) provides a more intuitive idea of negation than “! bound(x)”. Give a general pattern how to express (P_1 MINUS P_2) in SPARQL 1.0 syntax.

d) Recall the definition of ⟕ in the relational algebra (DB lecture) and define SPARQL’s ⟕ in a similar way.

(Parts of the solution are taken from [PAG06]: Jorge Pérez, Marcelo Arenas, Claudio Gutierrez: Semantics and Complexity of SPARQL. International Semantic Web Conference 2006: 30-43, and from [AG08]: Renzo Angles and Claudio Gutierrez: The Expressive Power of SPARQL. International Semantic Web Conference 2008: 114-129; use `http://www.dblp.org`)

a) Consider $R(A, B, C)$ and $S(A, B, D)$ where A is non-null, and B can contain nulls. Then, the null-tolerant join $\bowtie_{null}$ can be defined by the following steps:

1) cartesian product of both relations, and immediately evaluating the condition

$$R.A = S.A \wedge (R.B = S.B \vee (R.B \text{ is null}) \vee (S.B \text{ is null})) \ .$$

The result has the format $[R.A, S.A, R.B, S.B, C, D]$.

- $R.A$ has always the same (non-null) value as $S.A$.
- $R.B$ and $S.B$ can contain the same non-null-value, but also any of them can contain a null value, while the other is also null, or contains a non-null value.

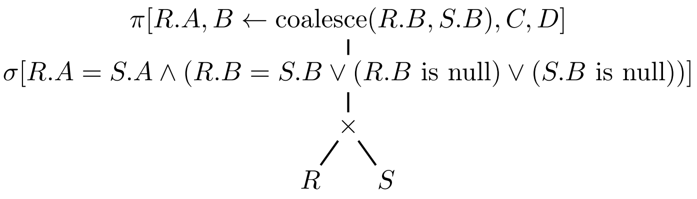
2) apply a projection that removes $S.A$.

3) For handling B, a new basic operator has to be defined (similar to SQL’s binary “coalesce” function: if the first argument is null, take the second one):

$$\text{coalesce} : \mathit{ANY} \times \mathit{ANY}, \qquad \begin{array}{ll} (v_1, v_2) \mapsto v_1 & \text{if } v_1 \text{ is not null,} \\ (\mathit{null}, v) \mapsto v & \end{array}$$

(note that coalesce$(R.B, S.B)$ = coalesce$(S.B, R.B)$ after evaluating the condition in Step (1)).

The algebra expression is then


$\pi[R.A, B \leftarrow \text{coalesce}(R.B, S.B), C, D]$
$\sigma[R.A = S.A \wedge (R.B = S.B \vee (R.B \text{ is null}) \vee (S.B \text{ is null}))]$
$\times$
R
S

b) SQL’s “`WHERE NOT EXISTS ...`” is similar.
Consider $R \setminus S$ with R and S as above.

```
SELECT * FROM R WHERE NOT EXISTS (
      SELECT * FROM S
      WHERE R.A = S.A AND (R.B = S.B OR R.B is null OR S.B is null)).
```

c) (taken from [AG08], Section 3)
The basic idea is to replace (P_1 MINUS P_2) by

$$((P_1 \text{ OPT } P_2) \text{ FILTER } (!(\text{bound}(?Y))))$$

where Y is a variable that occurs in P_2, but not in P_1.

Two more aspects have to be considered:

- If P_2 is of the form (P_2' OPT P_2''), then Y must be a variable from P_2' – i.e., a *non-optional variable* (otherwise there are solutions to P_2 that do not bind it).
- If there is no such variable (i.e. all non-optional variables of P_2 occur also in P_1), one must introduce one: take any non-optional triple pattern T that
  i) contains at least one new variable X' and
  ii) is sure to be satisfied whenever (P_1 and) P_2 is satisfied (i.e., it can be a renamed copy ($?X$ q $?X'$) of some triple pattern ($?X$ q $?Z$) from P_1, or any arbitrary pattern that is known to be satisfied in the application)

  and use $((P_1 \text{ OPT } (P_2 \text{ AND } T)) \text{ FILTER } (!(\text{bound}(X'))))$.

Example: Names of countries with their cities, where the city is not the capital:

```
# cities-not-capital.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
select ?X ?C
from <file:mondial.n3>
where { ?X a mon:Country; mon:hasCity ?C
        OPTIONAL {?X mon:capital ?C . ?X mon:capital ?C2}
        FILTER (!(bound(?C2))) }
```

Variables X and C occur in P_1 and in P_2, so a (useless) triple pattern is added to bind C_2.

d) $\Omega_1 ⟕ \Omega_2 = (\Omega_1 \bowtie \Omega_2) \cup (\Omega_1 \setminus_s (\Omega_1 \ltimes \Omega_2))$ where the semijoin is defined as usual as $\Omega_1 \ltimes \Omega_2 = \pi[\text{var}(\Omega_1)](\Omega_1 \bowtie \Omega_2)$, and $\setminus_s$ denotes the classical set difference from the relational algebra.

Note that it is not necessary to extend the second part of the union with null values (which must be done in the relational algebra to have the same format on both sides of the union).

Exercise 2.2 (Outer Join) Recall that SPARQL's OPTIONAL corresponds to a left outer join.

a) Give a general pattern how to express a *full* outer join (i.e., "outer" to both sides) in the SPARQL algebra (consider as input two mappings R and S and give an expression for $R ⟗ S$) and in SPARQL.

b) Give all cities (name as ?XN) that are the capital of a country (:capital) or that are located at a river (:locatedAt) or both (return the names ?CN of the country and/or the river (?RN)).

a) Replace the full outer join by a two left outer joins: $(R ⟕ S) \cup (S ⟕ R)$. Note that the intersection of both subterms is the inner join. With set semantics, these duplicates are automatically removed. Otherwise apply a DISTINCT.

With multiset semantics (or in the SPARQL query language), the inner join must be removed from the second term:

$$\begin{array}{rl} & (R ⟕ S) \cup ((S ⟕ R) \setminus_s (S \bowtie R)) \\ \text{or} & (R ⟕ S) \cup ((S ⟕ R) \setminus (S \ltimes R)) \end{array}$$

(recall that $\setminus$ denotes the not-exists-like operator from the SPARQL algebra, and $\setminus_s$ denotes the classical set difference).

For SPARQL, the query is of the form

```
    DISTINCT ... WHERE { { P_R(X) OPTIONAL P_S(Y) }
                         UNION
                         { P_S(Y) OPTIONAL P_R(X) } }
  or
    ... WHERE { { P_R(X) OPTIONAL P_S(Y) }
                UNION
                { { P_S(Y) OPTIONAL P_R(X) } FILTER (!(bound(X))) } }
```

b) There is an intuitive solution that replaces the outer join by two optionals: take a city, and if it is a capital, list the country, and if it is located at a river, list the river (multiple answers if it is located at several rivers):

```
# capitals-at-rivers-1.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
select ?XN ?CN ?RN
from <file:mondial.n3>
where { ?X a mon:City ; mon:name ?XN.
        OPTIONAL { ?C a mon:Country; mon:name ?CN; mon:capital ?X }
        OPTIONAL { ?X mon:locatedAt ?R . ?R a mon:River; mon:name ?RN }
        ## FILTER (bound(?C) || bound(?R)) }
```

The query yields at least one line for *each* city, including those that are neither capitals, nor located at some river(s). These can be removed by adding
`FILTER (bound(?C) || bound(?R))`.

The second solution applies the solution of (a):

```
# capitals-at-rivers-2.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
select ?XN ?CN ?RN
from <file:mondial.n3>
where { { ?X a mon:City ; mon:name ?XN .
          ?C a mon:Country; mon:name ?CN; mon:capital ?X
          OPTIONAL { ?X mon:locatedAt ?R . ?R a mon:River; mon:name ?RN } }
        UNION
        { ?X a mon:City ; mon:name ?XN .
          ?X mon:locatedAt ?R . ?R a mon:River; mon:name ?RN
          OPTIONAL { ?C a mon:Country; mon:name ?CN; mon:capital ?X }
          FILTER (!bound(?C)) }
      }
```

Exercise 2.3 (SPARQL Formal Semantics: OPTIONAL) Consider the SPARQL Formal Semantics.

Prove or show a counterexample:

The statement (from W3C SPARQL Working Draft 20061004)

> If OPT(A, B) is an optional graph pattern, where A and B are graph patterns, then S is a solution of OPT(A,B) if
>
> - S is a pattern solution of A and of B, or
> - S is a solution to A, but not to A and B.

describes the same semantics as above.

The given characterization is the one from the W3C SPARQL Recommendation from 20061004.

This "definition" is not only lacking formality, it reads as a weak definition anyway. E.g., it is missing "S is a solution to A which *cannot be extended with a solution to B* to a solution of A and B" which is required for the left outer join in the relational algebra.

How to "break" it? (This is not RDF-specific, but would work in the same way with SQL outer joins).

The counterexample that illustrates that central problem by showing that this does not only add "incomplete" final results, but as a consequence of "implicitly allowing" incomplete subresult tuples, yields also *wrong final answers* when having nested patterns is taken from [PAG06], Examples 1 and 3:

Consider the RDF database D:

$$
\begin{array}{llllllll}
D = \{ & (B_1 & \text{name} & \text{paul}), & (B_1 & \text{phone} & \text{777-3426}), & \\
 & (B_2 & \text{name} & \text{john}), & (B_2 & \text{email} & \text{john@acd.edu}), & \\
 & (B_3 & \text{name} & \text{george}), & (B_3 & \text{webPage} & \text{www.george.edu}), & \\
 & (B_4 & \text{name} & \text{ringo}), & (B_4 & \text{email} & \text{ringo@acd.edu}), & \\
 & (B_4 & \text{webPage} & \text{www.starr.edu}), & (B_4 & \text{phone} & \text{888-4537}) & \}
\end{array}
$$

Query pattern:

$$P = ((?X,\text{name, paul})\ \text{OPT}\ ((?Y,\ \text{name, george})\ \text{OPT}\ (?X,\ \text{email},\ ?Z))) =: (P_1\ \text{OPT}\ (P_2\ \text{OPT}\ P_3))\ .$$

The inner OPT is actually a "weak" cartesian product, which -according to the textual definition- is satisfied also by "left-only, right-null" answers.

$[\![P_1]\!] = \{\{X/B_1\}\}$.
$[\![P_2]\!] = \{\{Y/B_3\}\}$.
$[\![P_3]\!] = \{\{X/B_2, Z/\text{john@}\},\ \{X/B_4, Z/\text{ringo@}\}\}$.
$[\![P_2\ \text{OPT}\ P_3]\!] = [\![P_2 \sqsupset\!\!\bowtie P_3]\!] = \{\{X/B_2, Y/B_3, Z/\text{john@}\},\ \{X/B_4, Y/B_3, Z/\text{ringo@}\}\}$.
$[\![P]\!] = [\![P_1]\!] \sqsupset\!\!\bowtie [\![P_2 \sqsupset\!\!\bowtie P_3]\!] = \{\{X/B_1\}\}$.

On the other hand according to the textual W3C characterization, $S := \{\{X/B_1, Y/B_3\}\}$ is a solution to P: $S := \{\{X/B_1, Y/B_3\}\}$ is a solution to P_1 and to P_2 OPT P_3; the latter holds since it is a solution to P_2, although not to P_3.

The counterexample exploits the fact that it is not *well-designed* (i.e., X occurs inside the inner optional, and in the outermost pattern, but not directly outside the inner optional).

Note that the "declarative", but non-algebraic W3C characterization is also problematic from the operational aspect since the solution must first be guessed before being tested. An algebraic (and thus compositional) semantics allows a bottom-up computation from inside-out.

Exercise 2.4 (SPARQL: Filter-Safe Expressions) Consider the following definition:

Definition 1 ([PAG06, AG 08]) A SPARQL expression is *filter-safe*, if for every subexpression of the form $(P\ \ \text{FILTER}\ \ R)$, $\text{var}(R) \subseteq \text{var}(P)$.

a) Give a SPARQL query for the following: For each country, give the capital and its population, and, if exist, all cities in that country that have a higher population, as a result table of the following form

C	CAP	CapPop	City	CityPop
D	Berlin	3472009		
CH	Bern	128848	Zürich	384786
CH	Bern	128848	Genf	191557
:	:	:	:	

b) Make your query filter-safe (if it is not yet filter-safe).

c) Give the same query in SQL (hint: use the LEFT OUTER JOIN ... ON ... construct)

d) Sketch an algorithm that rewrites non-filter safe queries into safe ones. First, try it on your own, then maybe look in [AG08].

e) Give a SPARQL query for “Give the names of all countries, such that there is some city in that country where more than 1/4 of the population are living in”, and make it filter-safe.

f) Give a SPARQL query for “Give the names of all countries, such that there is *no* city in that country where more than 1/4 of the population are living in”, and make it filter-safe.

g) Is there a similar thing in SQL, in the relational algebra, and in the relational calculus (DB Theory/Deductive DB Lecture)?

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a)
```
# country-cap-biggercities.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
select ?C ?N1 ?P1 ?N2 ?P2
from <file:mondial.n3>
where { ?X a mon:Country; mon:carCode ?C; mon:capital ?CAP .
        ?CAP mon:name ?N1; mon:population ?P1 .
        OPTIONAL {  ?X mon:hasCity [ mon:name ?N2; mon:population ?P2 ]
                   FILTER (?P2 > ?P1) }}
```

b) An intuitive solution is to duplicate (relevant parts of) the pattern outside the OPTIONAL for binding of the variable to inside:

```
# country-cap-biggercities-safe.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
select ?C ?N1 ?P1 ?N2 ?P2
from <file:mondial.n3>
where { ?X a mon:Country; mon:carCode ?C; mon:capital ?CAP .
        ?CAP mon:name ?N1; mon:population ?P1 .
        OPTIONAL { ?X a mon:Country; mon:capital ?CAP .
                   ?CAP mon:population ?P1 .
                   ?X mon:hasCity [ mon:name ?N2; mon:population ?P2 ]
                   FILTER (?P2 > ?P1) }}
```

The (copied) variables occurring in the inner pattern and in the outer pattern act as join variables when computing the OPTIONAL-outer join.

Note that copying `?X a :Country` also inside the OPTIONAL might make the evaluation more efficient by restricting ?X to countries (in contrast to anything that has a population).

---

c)
```
SELECT x.code, cap.name, cap.population, y.name, y.population
FROM city cap,
   country x
   LEFT OUTER JOIN
   ( SELECT code, c.name, c.population
     FROM country x2, city cap2, city c
     WHERE x2.capital= cap2.name AND x2.code = cap2.country
       AND x2.province = cap2.province
       AND c.country = x2.code
       AND c.population > cap2.population ) y
   ON x.code = y.code
WHERE x.capital= cap.name AND x.code = cap.country AND x.province = cap.province
```

Note the structure of the FROM line:

- the left outer join is between country and the nested SFW,
- city cap is only then joined with the result (for the output).

Similar to the SPARQL filter-safe expression, the capital's population must be accessed inside the nested SFW which forms the right-hand-side of the left outer join.

d) The complete algorithm can be found in [AG08]. There, the fourth if-case (lines 7-10) covers the above case. The fifth if-case (lines 12-24) just throws an error in case a filter is still unsafe (having *bound*(c) for a constant results in an error), otherwise replaces *bound*(X) by false because it cannot be bound.

Note (DB Theory/Deductive DB lecture): consider the similarity with the transformation of general formulas into RANF when moving conjuncts into negated subformulas to make them *self-contained*.

e) The solution is of the same form as above:

```
# quarterpop.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
SELECT distinct ?N
FROM <file:mondial.n3>
WHERE {?X a mon:Country; mon:name ?N; mon:population ?XP .
       OPTIONAL { ?X mon:hasCity ?C . ?C mon:population ?CP .
                  FILTER (?CP > 0.25 * ?XP) } .
       FILTER (BOUND(?C)) }
```

and its filter-safe variant is

```
# quarterpop-safe.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
SELECT distinct ?N
FROM <file:mondial.n3>
WHERE {?X rdf:type mon:Country; mon:name ?N; mon:population ?XP .
       OPTIONAL { ?X mon:hasCity ?C; mon:population ?XP .
                  ?C mon:population ?CP .
                  FILTER (?CP > 0.25 * ?XP) } .
       FILTER (BOUND(?C)) }
```

f) The solution is again of the same form as above:

```
# noquarterpop.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
SELECT distinct ?N
FROM <file:mondial.n3>
WHERE {?X a mon:Country; mon:name ?N; mon:population ?XP .
       OPTIONAL { ?X mon:hasCity ?C . ?C mon:population ?CP .
                  FILTER (?CP > 0.25 * ?XP) } .
       FILTER (!BOUND(?C)) }
```

and its filter-safe variant is

```
# noquarterpop-safe.sparql
prefix mon: <http://www.semwebtech.org/mondial/10/meta#>
prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#>
SELECT distinct ?N
FROM <file:mondial.n3>
WHERE {?X rdf:type mon:Country; mon:name ?N; mon:population ?XP .
       OPTIONAL { ?X mon:hasCity ?C; mon:population ?XP .
                  ?C mon:population ?CP .
                  FILTER (?CP > 0.25 * ?XP) } .
       FILTER (!BOUND(?C)) }
```

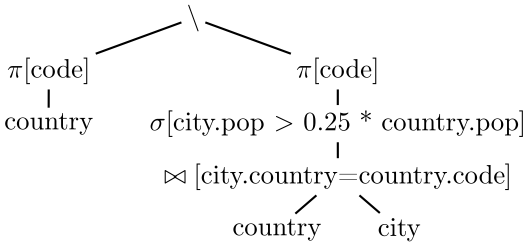
g) Yes: the same issue occurs typically with correlated subqueries (when translating positive correlated subqueries into joins/semijoins, and when translating negated (NOT EXISTS) subqueries into MINUS or into the safe relational calculus (for positive ones, the logical join variables in the scope of the outer quantifier can do the work!)).

The SQL OUTER JOIN also join requires also to make the the right side *self-contained* (in the SQL code in Exercise 1.2 the country table has also to be added to the right side of the outer join to be able to access country.population).

Example. Consider the NOT EXISTS examples from above. Its SQL equivalent is

```
SELECT name
FROM country
WHERE NOT EXISTS
  (SELECT *
   FROM city
   WHERE city.country= country.code
     AND city.population > 0.25 * country.population)
```

The equivalent relational algebra query requires rewriting into a minus:


\
π[code]
π[code]
country
σ[city.pop > 0.25 * country.pop]
⋈ [city.country=country.code]
country
city

The access to the country table must be added inside the right-hand-side.

(This requires background knowledge from the DB Theory/Deductive DB lecture)

The query in the relational calculus is

$$\begin{aligned} Q(C) = \; & \exists N, Pop, Area, Cap, CapProv: \\ & \mathsf{country}(N, C, Pop, Area, Cap, CapProv) \wedge \\ & \neg\exists CN, CProv, CPop, Long, Lat, El : (\mathsf{city}(CN, C, CProv, CPop, Long, Lat, El) \wedge CPop > 0.25 * Pop) \end{aligned}$$

which is not in RANF (there is no positive occurrence of Pop inside the not-exists. Its RANF form is obtained by push-into-not-exists:

$$\begin{array}{rll} Q'(C) = & \exists N, Pop, Area, Cap, CapProv : & \\ & \mathsf{country}(N, C, Pop, Area, Cap, CapProv) \land & \\ & \neg\exists CN, CProv, CPop, Long, Lat, El : & (\mathsf{country}(N, C, Pop, Area, Cap, CapProv) \land \\ & & (\mathsf{city}(CN, C, CProv, CPop, Long, Lat, El) \land CPop > 0.25 * Pop) \end{array}$$

The RANF form is also required for translating into Datalog.