Process Modeling with Different Qualities of Knowledge

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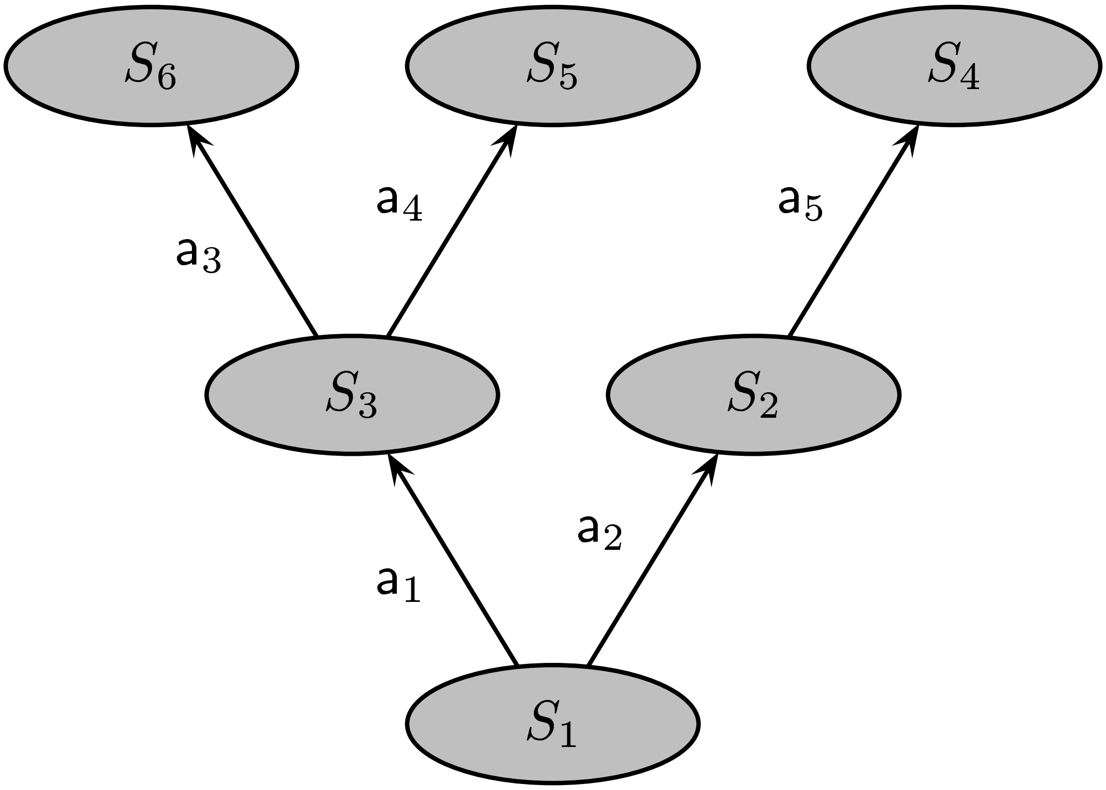
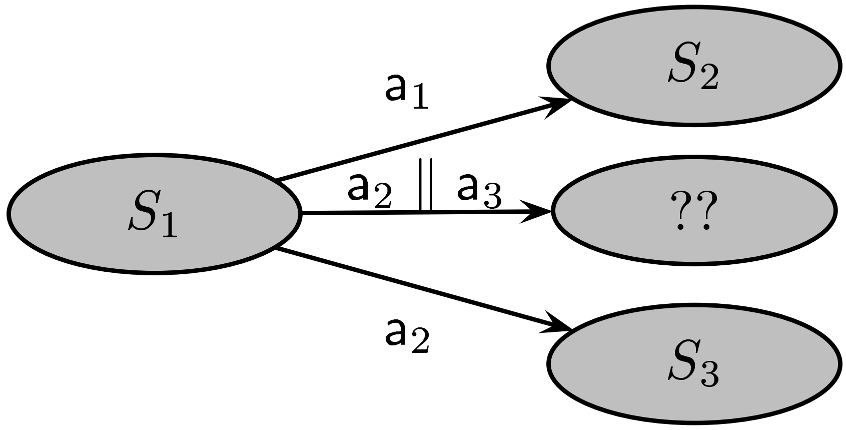
Process Modeling

States + Actions

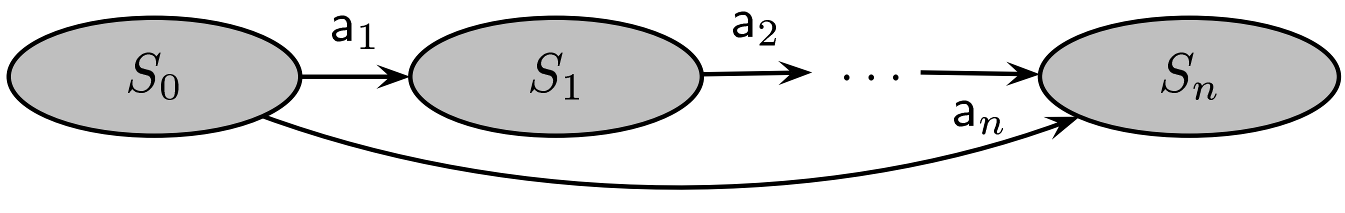


Problems

Parallel Composition



Sequential Composition



$$\mathsf{a}_1 \cdot \mathsf{a}_2 \cdot \ldots \cdot \mathsf{a}_n = ??$$

Labeled Kripke-Structures

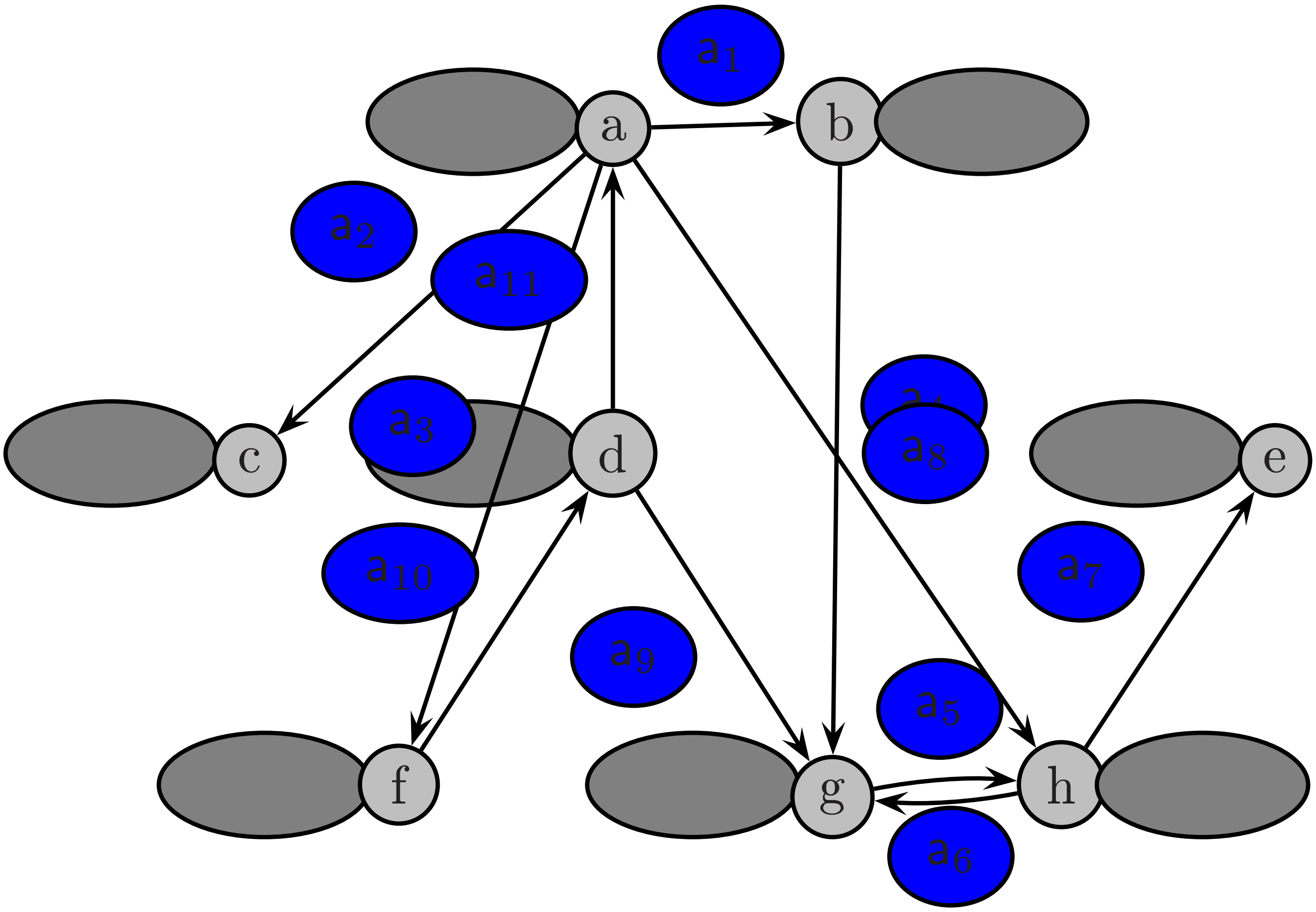
$$\mathbf{K} = (\mathbf{G}, \mathbf{A}, \mathbf{R}, \mathbf{M})$$

G: A set of states (names),

A: A set of actions,

$\mathbf{R} \subset \mathbf{G} \times 2^{\mathbf{A}} \times \mathbf{G}$: An accessibility relation,

M: $\mathbf{G} \to$ {First-order interpretations over a signature Σ}



The Language of Actions

- Signature Σ_A of action symbols,
- For $a \in \Sigma_A$ and $u_1, \ldots, u_n \in \mathbf{U}$, $a(u_1, \ldots, u_n)$ is an elementary action.
- For actions $\mathsf{a}, \mathsf{a}_1, \mathsf{a}_2$,

  $\mathsf{a}_1 \parallel \mathsf{a}_2$ (parallel),

  $\mathsf{a}_1 \otimes \mathsf{a}_2$ (sequential),

  $\mathsf{a} \vee \mathsf{b}$ (alternative),

  a^ω (iteration), and

  $\mathsf{a}^\star$ (fixpoint)

  are composite actions.
- For an action a with arguments $(u_1, \ldots, u_n)$ and an action symbol p,
  $\mathsf{p} = p(u_1, \ldots, u_n) := \mathsf{a}$, is a *procedure definition*. p is called a *procedure*.

Transitions

$$\mathsf{a}(\mathbf{I}) = \{\mathbf{J} \mid \text{application of } \mathsf{a} \text{ in } \mathbf{I} \text{ can result in } \mathbf{J}\,\}$$

(Partial) First-Order Interpretations

- Σ, $\mathbf{I}$, $\mathbf{U} \ni u_1, \ldots, u_n$
- f: function symbol, p: predicate symbol
- $f(u_1, \ldots, u_n)$: *function application*
- $p(u_1, \ldots, u_n)$: *predicate application*

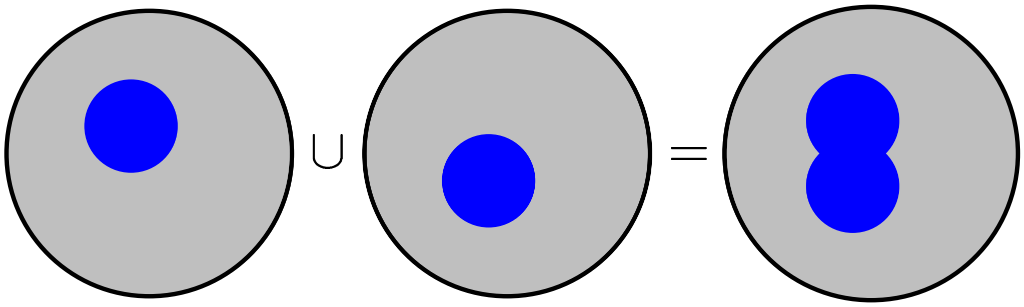
domain:

$$\begin{aligned}\mathsf{dom}(\mathbf{I}) := \; & \{(p; u_1, \ldots, u_m) \mid \\ & \quad (I(p))(u_1, \ldots, u_m) \text{ is defined}\} \;\cup \\ & \{(f; u_1, \ldots, u_n) \mid \\ & \quad (I(f))(u_1, \ldots, u_n) \text{ is defined}\} \quad .\end{aligned}$$

Operations on Partial First-Order Interpretations

union $\mathbf{J} := \mathbf{I} \cup \mathbf{I}'$:

$$(J(p))(u_1, \ldots, u_n) :=$$
$$\sqcup((I(p))(u_1, \ldots, u_n), (I'(p))(u_1, \ldots, u_n))$$
$$(J(f))(u_1, \ldots, u_m) :=$$
$$\sqcup((I(f))(u_1, \ldots, u_m), (I'(f))(u_1, \ldots, u_m)) \quad .$$

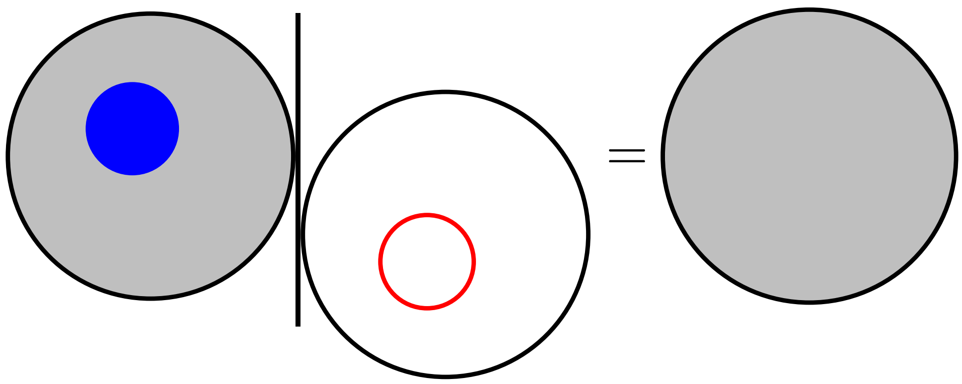


Operations on Partial First-Order Interpretations

restriction $\mathbf{I}|_R = (I|_R, \mathbf{U})$ of $\mathbf{I}$ to $R := F \cup P$:

$$(I|_R\,(p))(u_1,\ldots,u_n) := \begin{cases} (I(p))(u_1,\ldots,u_n) & \text{if } (p;u_1,\ldots,u_n) \in P\ , \\ \perp & \text{otherwise, and} \end{cases}$$

$$(I|_R\,(f))(u_1,\ldots,u_n) := \begin{cases} (I(f))(u_1,\ldots,u_n) & \text{if } (f;u_1,\ldots,u_n) \in F\ , \\ \perp & \text{otherwise} \quad . \end{cases}$$



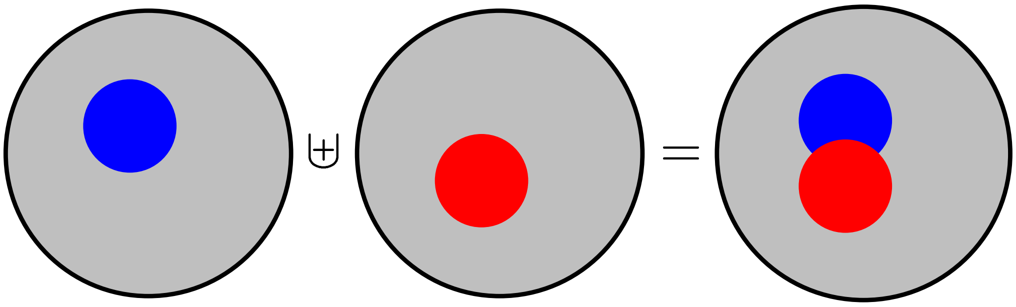
Operations on Partial First-Order Interpretations

superposition $\mathbf{J} = (J, \mathbf{U}) = \mathbf{I} \uplus_R \mathbf{I}'$ of $\mathbf{I}$ with $\mathbf{I}'$ on $R := F \cup P$:

$$(J(p))(u_1, \ldots, u_n) := \begin{cases} (I'(p))(u_1, \ldots, u_n) & \text{if } (p; u_1, \ldots, u_n) \in P \quad , \\ (I(p))(u_1, \ldots, u_n) & \text{otherwise, and} \end{cases}$$

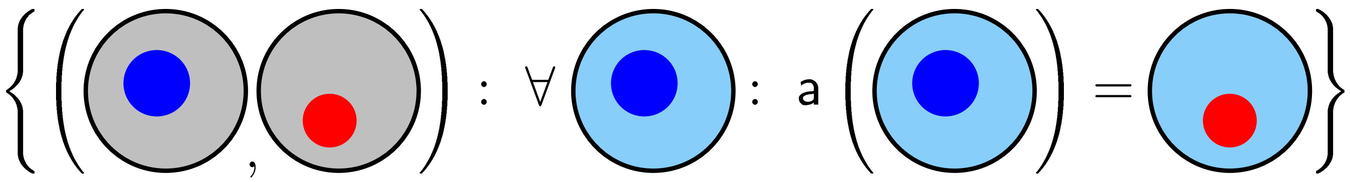
$$(J(f))(u_1, \ldots, u_n) := \begin{cases} (I'(f))(u_1, \ldots, u_n) & \text{if } (f; u_1, \ldots, u_n) \in F \quad , \\ (I(f))(u_1, \ldots, u_n) & \text{otherwise} \quad . \end{cases}$$

$\mathbf{I} \uplus \mathbf{I}' := \mathbf{I} \uplus_{\mathsf{dom}(\mathbf{I}')} \mathbf{I}'$.

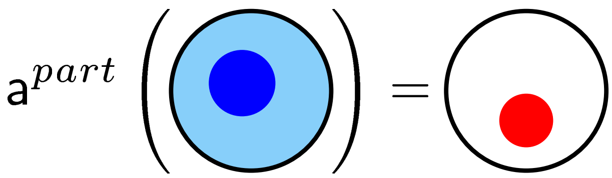


Specification

$$\begin{aligned}\mathsf{Spec}(\mathsf{a}) := \{(\mathbf{P},\mathbf{Q}) \in \mathbb{I}^p_\Sigma \times \mathbb{I}^p_\Sigma \mid \; & \\ & \text{for every } \mathbf{I} \in \mathbb{I}_\Sigma \text{ with } \mathbf{I} \supseteq \mathbf{P}, \\ & \mathsf{a} \text{ is executable in } \mathbf{I} \text{ and } \mathbf{I} \uplus_{\mathsf{dom}(\mathbf{Q})} \mathbf{Q} \in \mathsf{a}(\mathbf{I})\} \;.\end{aligned}$$



$$\mathsf{a}^{part}(\mathbf{I}) := \{\mathbf{Q} \mid (\mathbf{P},\mathbf{Q}) \in \mathsf{Spec}(\mathsf{a}) \text{ and } \mathbf{P} \subseteq \mathbf{I}\} \;.$$



a^{part} contains *only* the guaranteed knowledge.

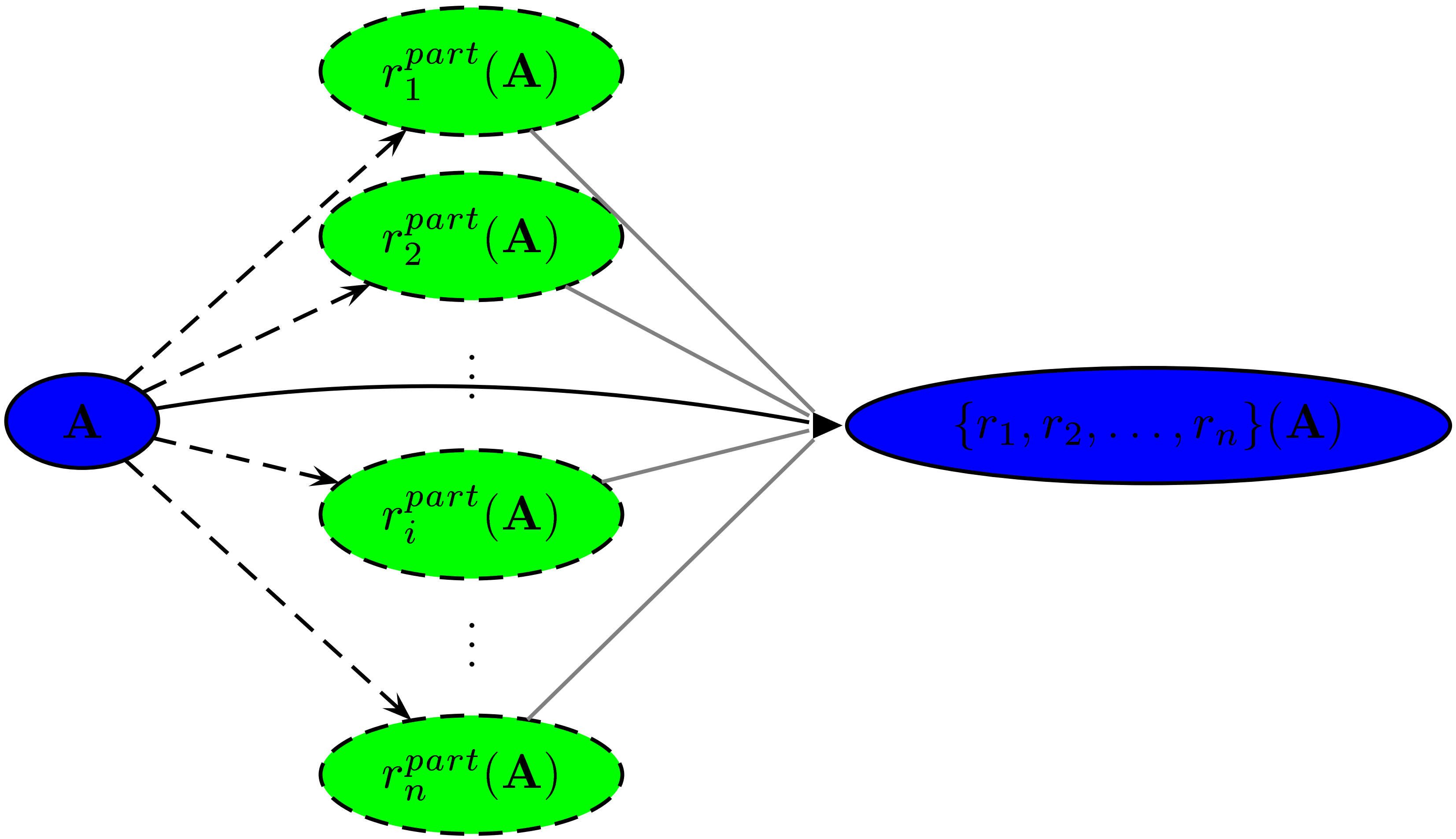
Composite Actions

$$\mathsf{Spec}(\mathsf{a} \parallel \mathsf{b}) =$$
$$\{(\mathbf{P},\mathbf{Q}) \mid \mathbf{P},\mathbf{Q} \in \mathbb{I}^p_\Sigma \text{ and there are } (\mathbf{P}_\mathsf{a},\mathbf{Q}_\mathsf{a}) \in \mathsf{Spec}(\mathsf{a}),$$
$$(\mathbf{P}_\mathsf{b},\mathbf{Q}_\mathsf{b}) \in \mathsf{Spec}(\mathsf{b}) \text{ s.t. } \mathbf{P} = \mathbf{P}_\mathsf{a} \cup \mathbf{P}_\mathsf{b} \text{ and } \mathbf{Q} = \mathbf{Q}_\mathsf{a} \cup \mathbf{Q}_\mathsf{b}\}\ ,$$

$$\mathsf{Spec}(\mathsf{a} \otimes \mathsf{b}) =$$
$$\{(\mathbf{P},\mathbf{Q}) \mid \mathbf{P},\mathbf{Q} \in \mathbb{I}^p_\Sigma \text{ and there are } (\mathbf{P}_\mathsf{a},\mathbf{Q}_\mathsf{a}) \in \mathsf{Spec}(\mathsf{a}),$$
$$(\mathbf{P}_\mathsf{b},\mathbf{Q}_\mathsf{b}) \in \mathsf{Spec}(\mathsf{b}) \text{ s.t. } \mathbf{Q}_\mathsf{a} \cup \mathbf{P}_\mathsf{b} \text{ is consistent and}$$
$$\mathbf{P} = \mathbf{P}_\mathsf{a} \cup \mathbf{P}_\mathsf{b}|_{\mathsf{dom}(\mathbf{P}_\mathsf{b})\setminus\mathsf{dom}(\mathbf{Q}_\mathsf{a})} \text{ and } \mathbf{Q} = \mathbf{Q}_\mathsf{a} \uplus \mathbf{Q}_\mathsf{b}\}\ ,$$

$$\mathsf{Spec}(\mathsf{a} \vee \mathsf{b}) = \mathsf{Spec}(\mathsf{a}) \cup \mathsf{Spec}(\mathsf{b}) \cup \mathsf{Spec}(\mathsf{a} \parallel \mathsf{b})\ ,$$

State Space with Partial Interpretations



Kripke Structures with Elementary Parallel Changes

$\mathbf{K} = (\mathbf{G}, \mathbf{G}_p, \mathbf{A}, \mathbf{R}, \mathbf{T}, \mathbf{S}, \mathbf{M})$

$\mathbf{G}$: set of *main states*,

$\mathbf{G}_p$: set of *auxiliary states*,

$\mathbf{A}$: set of *labels*, which are used to represent actions.

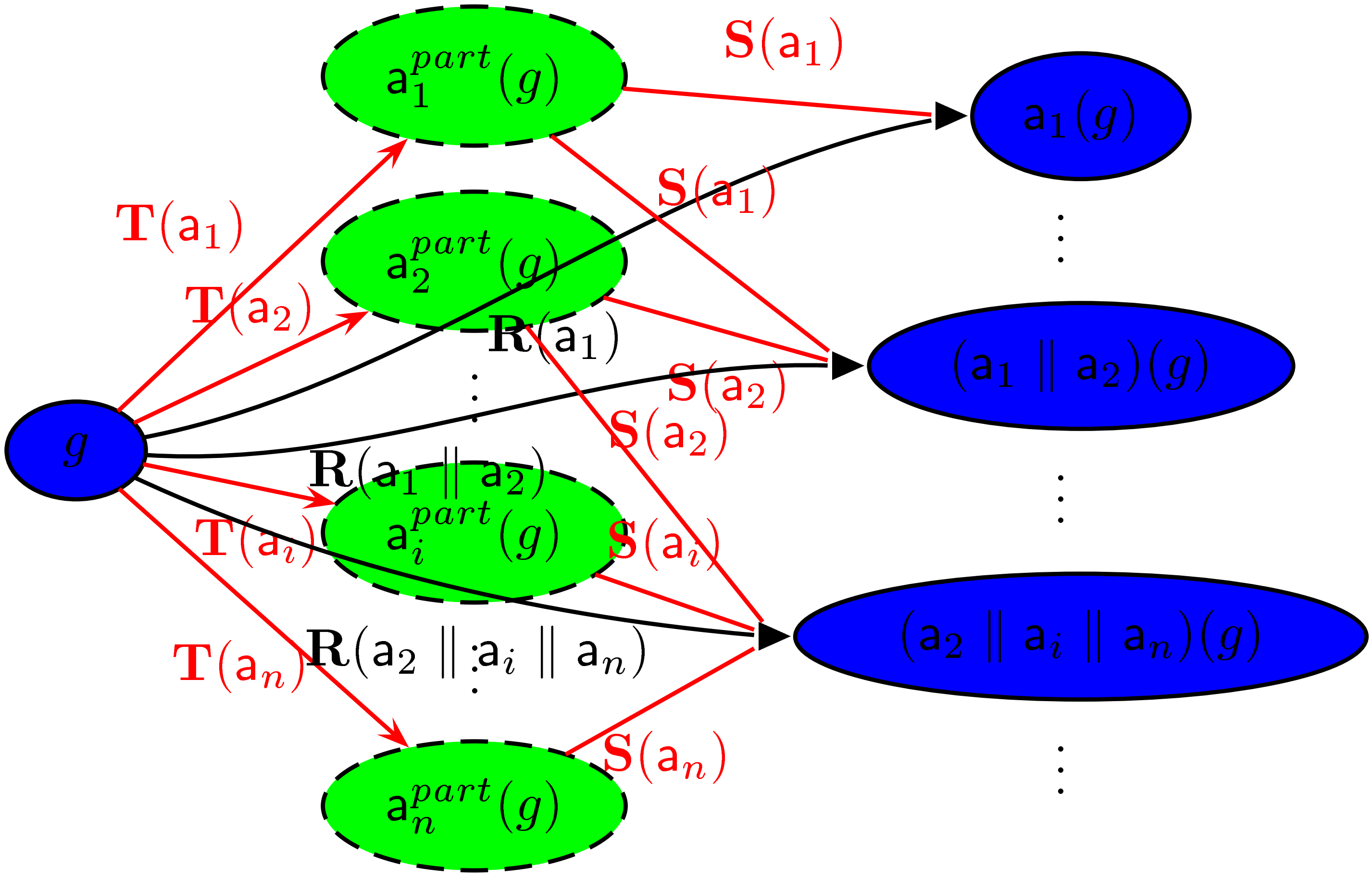
$\mathbf{R} \subseteq \mathbf{G} \times \mathbf{A} \times \mathbf{G}$: *temporal successor relation*,

$\mathbf{T} \subseteq \mathbf{G} \times \mathbb{A}_e \times \mathbf{G}_p$: *accessibility relation of auxiliary states from main states*,

$\mathbf{S} \subseteq \mathbf{G}_p \times \mathbb{A}_e \times \mathbf{G}$: *return relation from auxiliary states to main states*,

$\mathbf{M}$ is a mapping $\mathbf{G} \rightarrow \mathbb{I}$ and $\mathbf{G}_p \rightarrow \mathbb{I}^p$, giving an interpretation for every state.

Elementary parallel changes



Transition labeling:

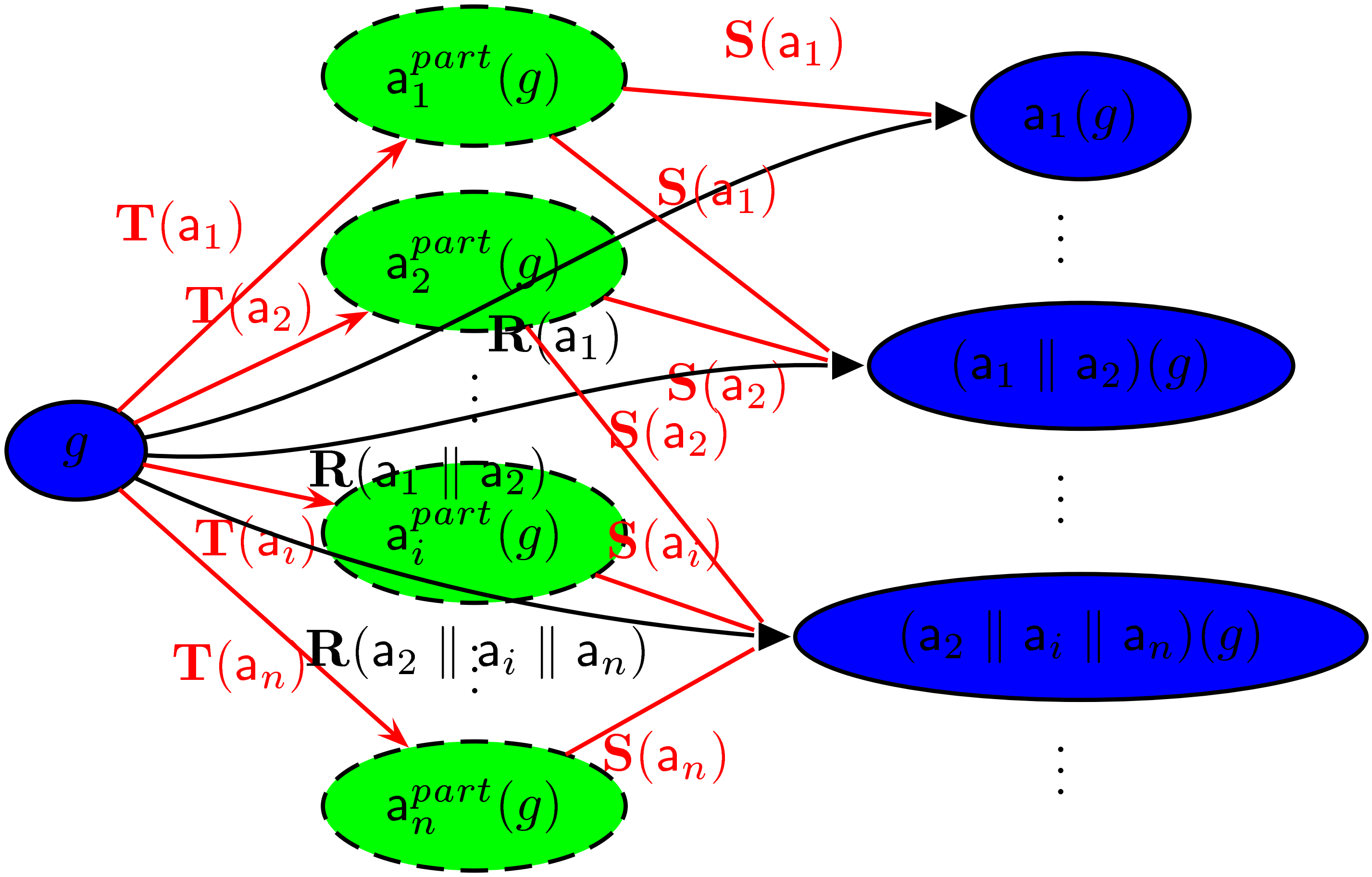
$$\mathbf{R}(g, \mathsf{a}_1[g_1] \parallel \ldots \parallel \mathsf{a}_k[g_k], h)$$

s.t. $\mathbf{T}(g, \mathsf{a}_i, g_i)$ and $\mathbf{S}(g_i, \mathsf{a}_i, h)$

Unambiguous if for every

$$\mathbf{S}(g, \mathsf{a}, h) \wedge \mathbf{S}(g', \mathsf{a}, h) \;\Rightarrow\; g = g' \;.$$

Elementary parallel changes



Set of input-states:

$$\mathbf{C}^{\leftarrow}(g, \mathsf{a}, h) :=$$
$$\{h' \mid \exists \mathsf{b} \in \mathbb{A}_e : \mathsf{b} \in_{\parallel} \mathsf{a} \wedge h' \in \mathbf{S}^{-1}(\mathsf{b}, h)\} \subset \mathbb{I}^p$$

Successor State:

$$\mathbf{C}(g, \mathsf{a}, h) = \mathbf{M}(g) \uplus \bigcup_{h' \in \mathbf{C}^{\leftarrow}(g,\mathsf{a},h)} \mathbf{M}(h') \ .$$

Problems

with this "state-space":

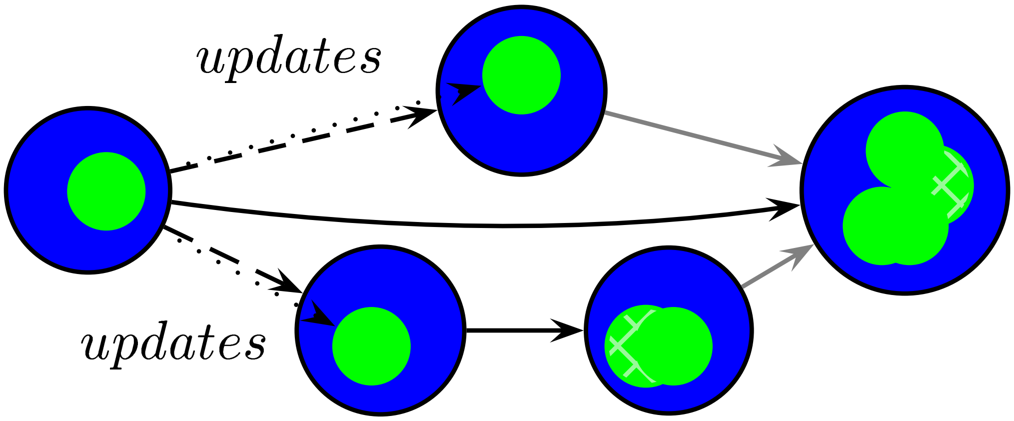
- Evaluating formulas in partial interpretations,
- Querying preconditions,
- Applying actions ?
- Frame-Problem.

Homogenous State Space

Two qualities of knowledge:

Total Interpretations: as-is knowledge

Partial Interpretations: safe, procedure knowledge



2-structure $\mathbf{I} = (\langle I \rangle_1, \langle I \rangle_2, \mathbf{U})$:

- $\langle \mathbf{I} \rangle_1 := (\langle I \rangle_1, \mathbf{U})$ total first-order interpretation,
- $\langle \mathbf{I} \rangle_2 := (\langle I \rangle_2, \mathbf{U})$ partial first-order interpretation,

and $\langle I \rangle_1 \supseteq \langle I \rangle_2$.

$$\mathsf{Spec}^2(\mathsf{a}) := \{((\mathbf{P}, \emptyset), (\emptyset, \mathbf{Q})) \mid (\mathbf{P}, \mathbf{Q}) \in \mathsf{Spec}(\mathsf{a})\}$$

$$\mathsf{a}(\langle I \rangle_1, \langle I \rangle_2, \mathbf{U}) = (\langle I \rangle_1 \uplus \mathbf{Q}, \langle I \rangle_2 \uplus \mathbf{Q}, \mathbf{U})$$

Hierarchical Labeled Kripke Structures

$\mathbf{K} = (\mathbf{G}, \mathbf{A}, \mathbf{R}, \mathbf{T}, \mathbf{Q}, \mathbf{S}, \mathbf{M})$

$\mathbf{G}$: set of *states*,

$\mathbf{A}$: set of *labels*,

$\mathbf{R} \subseteq \mathbf{G} \times \mathbf{A} \times \mathbf{G}$: *temporal successor relation*,

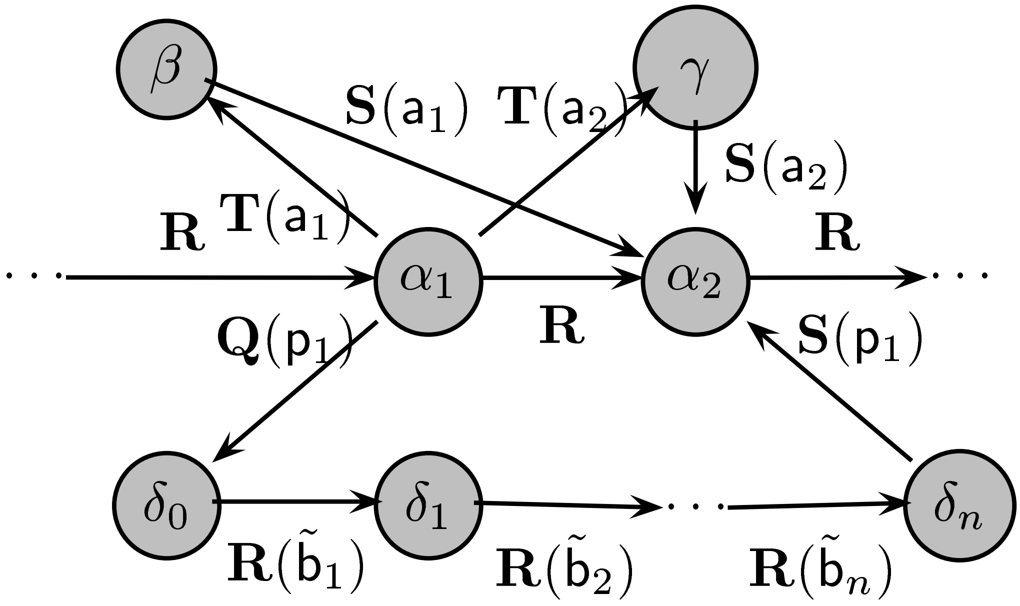
$\mathbf{T} \subseteq \mathbf{G} \times \mathbb{A}_e \times \mathbf{G}$: *accessibility relation for elementary actions*,

$\mathbf{Q} \subseteq \mathbf{G} \times \mathbb{A}_p \times \mathbf{G}$: *accessibility relation for procedure calls*,

$\mathbf{S} \subseteq \mathbf{G} \times (\mathbb{A}_p \cup \mathbb{A}_e) \times \mathbf{G}$: *labeled return relation* from states reached by elementary actions or executions of procedures.

$\mathbf{M} : \mathbf{G} \rightarrow \mathbb{I} \times \mathbb{I}_p$ maps states to 2-structures (i.e. $\langle \mathbf{M} \rangle_1 : \mathbf{G} \rightarrow \mathbb{I}$, $\langle \mathbf{M} \rangle_2 : \mathbf{G} \rightarrow \mathbb{I}_p$).

Hierarchical Kripke Structures



$$\mathbf{R}(\alpha_1, \mathsf{a}_1[\beta] \parallel \mathsf{a}_2[\gamma] \parallel \mathsf{p}_1[\delta_0, \tilde{\mathsf{b}}_1, \delta_1, \tilde{\mathsf{b}}_2, \ldots, \tilde{\mathsf{b}}_n, \delta_n], \alpha_2)$$

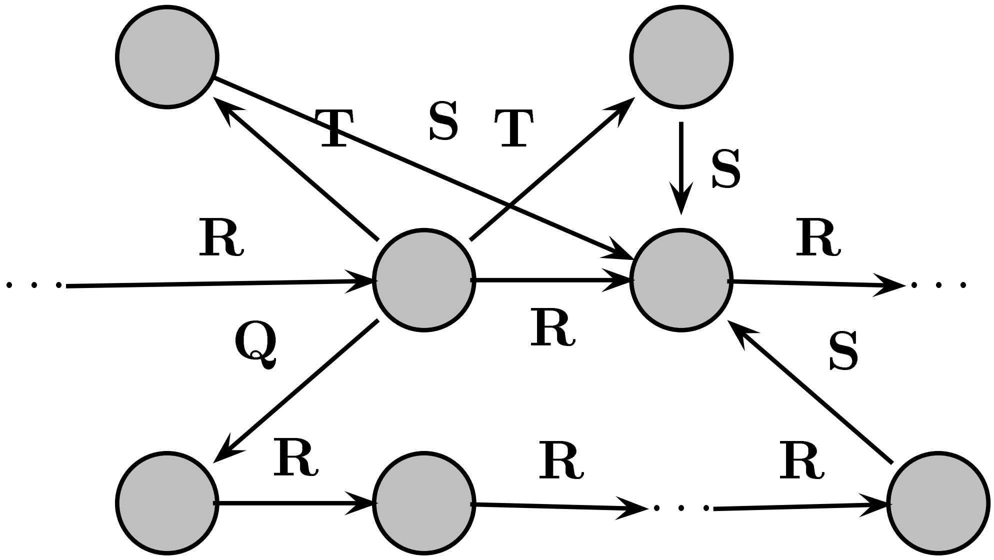
Consistency:

$$\mathbf{R}(g, \mathsf{a}, h) \wedge \mathbf{S}(h', \mathsf{b}, h) \wedge \mathsf{b} \in_{\parallel} \mathsf{a} \;\Rightarrow$$
$$((\mathsf{b} \in \mathbb{A}_e \;\Rightarrow\; \mathbf{T}(g, \mathsf{b}, h')) \wedge$$
$$(\mathsf{b} \in \mathbb{A}_p \;\Rightarrow\; \exists g' : \mathbf{Q}(g, \mathsf{b}, g') \wedge \mathbf{R}^*(g', h'))) \;,$$

Input States:

$$\mathbf{C}^{\leftarrow}(g, \mathsf{a}, h) =$$
$$\{h' \mid \exists \mathsf{b} \in \mathbb{A}_e \cup \mathbb{A}_p : \mathsf{b} \in_{\parallel} \mathsf{a} \wedge h' \in \mathbf{S}^{-1}(\mathsf{b}, h)\} \;.$$

Hierarchical Kripke Structures



Successor State:

$$\langle \mathbf{C} \rangle_2(g, \mathsf{a}, h) = \langle \mathbf{M} \rangle_2(g) \uplus \bigcup_{h' \in \mathbf{C}^{\leftarrow}(g,\mathsf{a},h)} \langle \mathbf{M} \rangle_2(h') \ ,$$
$$\langle \mathbf{C} \rangle_1(g, \mathsf{a}, h) = \langle \mathbf{M} \rangle_1(g) \uplus \langle \mathbf{C} \rangle_2(g, \mathsf{a}, h).$$

$$\begin{array}{lcl}
\mathbf{R}(g, \mathsf{a}, h) & \Rightarrow & \mathbf{M}(h) = \mathbf{C}(g, \mathsf{a}, h) \\
\mathbf{T}(g, \mathsf{a}, h), \mathsf{a} \in \mathbb{A}_e & \Rightarrow & \mathbf{M}(h) = \mathsf{a}(\langle M(g) \rangle_1, \emptyset, \mathbf{U}) \\
\mathbf{Q}(g, \mathsf{a}, h), \mathsf{a} \in \mathbb{A}_p & \Rightarrow & \mathbf{M}(h) = (\langle M(g) \rangle_1, \emptyset, \mathbf{U}) \ .
\end{array}$$

Axiomatization:

$\varphi \in \{f(u_1, \ldots, u_n),\ p(u_1, \ldots, u_n)\}.$

$$\frac{\mathbf{R}(g, \_, h) \quad , \quad \langle \mathbf{M}(g) \rangle_1 \vdash \varphi}{\langle \mathbf{M}(h) \rangle_1 \mathrel{|\!\sim} \varphi}$$

$$\frac{\mathbf{R}(g, \_, h) \quad , \quad \langle \mathbf{M}(g) \rangle_2 \vdash \varphi}{\langle \mathbf{M}(h) \rangle_2 \mathrel{|\!\sim} \varphi}$$

$$\frac{\mathbf{Q}(g, \_, h) \quad , \quad \langle \mathbf{M}(g) \rangle_1 \vdash \varphi}{\langle \mathbf{M}(h) \rangle_1 \mathrel{|\!\sim} \varphi}$$

$$\frac{\mathbf{T}(g, \_, h) \quad , \quad \langle \mathbf{M}(g) \rangle_1 \vdash \varphi}{\langle \mathbf{M}(h) \rangle_1 \mathrel{|\!\sim} \varphi}$$

$$\frac{\mathbf{T}(g, \mathsf{a}, h) \quad , \quad \mathsf{Spec}(\mathsf{a}), \mathbf{M}(g) \vdash \varphi}{\langle M(h) \rangle_2 \vdash \varphi}$$

$$\frac{\mathbf{S}(g, \_, \mathsf{a}, h) \quad , \quad \mathsf{a} \in \mathbf{A}_e \quad , \quad \langle \mathbf{M}(g) \rangle_2 \vdash \varphi}{\langle \mathbf{M}(h) \rangle_2 \vdash \varphi}$$

$$\frac{\langle \mathbf{M}(g) \rangle_2 \mathrel{|\!\sim} \varphi \quad , \quad \text{not} \quad \langle \mathbf{M}(g) \rangle_2 \vdash \neg\varphi}{\langle \mathbf{M}(g) \rangle_2 \vdash \varphi}$$

$$\frac{\langle \mathbf{M}(g) \rangle_2 \vdash \varphi}{\langle \mathbf{M}(g) \rangle_1 \vdash \varphi}$$

$$\frac{\langle \mathbf{M}(g) \rangle_1 \mathrel{|\!\sim} \varphi \quad , \quad \text{not} \quad \langle \mathbf{M}(g) \rangle_1 \vdash \neg\varphi}{\langle \mathbf{M}(g) \rangle_1 \vdash \varphi}$$

Application: Relational Active Database System

Language: Statelog; Datalog extended by states.
Operations on $r(x_1, \ldots, x_n)$:

$$\mathsf{ins}{:}r(x_1, \ldots, x_n),\ \mathsf{del}{:}r(x_1, \ldots, x_n).$$

$$[Z{+}1]\ R(\bar{X}) \leftarrow [Z]\ \mathsf{ins}{:}R(\bar{X}), \neg EOT.$$
$$[Z{+}1]\ R(\bar{X}) \leftarrow [Z]\ R(\bar{X}), \neg\mathsf{del}{:}R(\bar{X}), \neg EOT.$$

Updates are accumulated in protocol relations (= procedure knowledge)

$$[Z{+}1]\ \mathsf{insd}{:}R(\bar{X}) \leftarrow [Z]\ \mathsf{ins}{:}R(\bar{X}), \neg EOT.$$
$$[Z{+}1]\ \mathsf{insd}{:}R(\bar{X}) \leftarrow [Z]\ \mathsf{insd}{:}R(\bar{X}), \neg\mathsf{del}{:}R(\bar{X}), \neg EOT.$$
$$[Z{+}1]\ \mathsf{deld}{:}R(\bar{X}) \leftarrow [Z]\ \mathsf{del}{:}R(\bar{X}), \neg EOT.$$
$$[Z{+}1]\ \mathsf{deld}{:}R(\bar{X}) \leftarrow [Z]\ \mathsf{deld}{:}R(\bar{X}), \neg\mathsf{ins}{:}R(\bar{X}), \neg EOT.$$

Changes computed by procedures:

$$[Z]\ \mathsf{ins}{:}R(\bar{Y}) \quad \leftarrow \quad [Z]\ \pi(\bar{X}), [Z.\pi(\bar{X}).N]\ \mathsf{insd}{:}R(\bar{Y}), EOT.$$
$$[Z]\ \mathsf{del}{:}R(\bar{Y}) \quad \leftarrow \quad [Z]\ \pi(\bar{X}), [Z.\pi(\bar{X}).N]\ \mathsf{deld}{:}R(\bar{Y}), EOT.$$